

Sense and Meaning: The Origins and Development of the Distinction

Frege's distinction between sense (*Sinn*) and meaning (*Bedeutung*) is his most influential contribution to philosophy, however central it was to his own projects, and however he may have conceived its importance. Philosophers of language influenced by, or reacting against the distinction, and historians of philosophy commenting on it, have all contributed to the voluminous literature surrounding it.¹ Nonetheless in this essay I hope to shed new light on the distinction by considering it in the context of the development of Frege's thought, and connecting it more intimately than is usually done with Frege's interests in logic, especially his views on judgment, truth and inference, which were central to his own projects as he conceived them.

Frege does not employ the terminology of sense and meaning in his first great logical-philosophical work, the *Begriffsschrift* of 1879 (*BS*). However, *BS* already contains the seeds of the distinction in its notion of *content* (*Inhalt*). Tracing out the difficulties inherent in Frege's early talk of "content" illuminates the need for this distinction, as well as his further difficulties in formulating it. *BS* contains two distinct, yet interrelated, ancestors of the sense-meaning distinction. Section I discusses the first root of the distinction, which lies in Frege's notion of *judgeable content*, expressed by sentences. The second root lies in his account of identity sentences, and the associated idea of "modes of determination" of a content. Section II explores this account in detail, and reveals some of the difficulties inherent in it. Sections III and IV show how the needs of Frege's project in the philosophy of mathematics brought these difficulties to the fore and led to the development of the mature sense-meaning distinction. Sections V to VIII expound Frege's mature vision; section IX then examines some of the remaining difficulties in

the light of the development of the distinction.

I. *Begriffsschrift*: Judgeable Content

In the preface to *BS*, Frege set as his goal to determine the epistemological status of arithmetical truths. This requires investigating whether they can be proved on the basis of logical laws alone, or need some other source of support, such as Kantian pure intuition. To answer this question, he constructed a new logical system in which proofs could be carried out without “gaps,” so as to display explicitly all presuppositions and assumptions employed. In devising his logical notation he tried to express only that which is relevant to inference, which he called “conceptual content” (*begrifflicher Inhalt*). (*BS*, ix-x.) In §3 of *BS*, Frege explains this notion through an example: “At Plataea the Greeks defeated the Persians” and “At Plataea the Persians were defeated by the Greeks” have the same conceptual content, since the same *consequences* follow from each, in conjunction with any set of additional premises one might consider. (*BS* §3, 2-3.) Frege’s example suggests a more general principle. Letting ‘ \Rightarrow ’ represent an as yet unspecified relation of consequence:

(CONTENT): Sentences *A* and *B* have the same content if and only if, for any set of sentences (auxiliary premises) *S* and sentence (conclusion) *C*: $S, A \Rightarrow C$ if and only if $S, B \Rightarrow C$.

It may seem implausible to attribute such a general principle to Frege on the basis of an example used in an informal presentation. However, Frege is simply adapting a piece of traditional logical wisdom to his own purposes: the distinction between the extension and comprehension of an idea or concept. This goes back at least to the Port-Royal *Logic* (1662) and arguably much further, to Porphyry’s *Isagoge* and medieval commentators on it. For the Port-

Royalists, “the comprehension of an idea” consists in “the attributes that it *contains* in itself” (*Logic*, 39, my emphasis), in the sense in which the idea of *human* might be said to “contain” the idea of *animal*. This goes with a picture of ideas as arranged in a hierarchy, with some ideas containing others, higher in the hierarchy or tree. Logical relations of implication (etymologically, “folding-in”) are relations of containment of ideas – *human* contains *animal*, the first idea implies the second. On the other hand, ideas not only contain, but are contained in, other ideas. At the bottom of the tree we find the entities to which the ideas apply; these constitute the *extension* of the ideas. “I call the *extension* of an idea the subjects to which this idea applies.” (*Logic*, 40.)

In his *Logic*,² a work which Frege read, Kant applied this distinction to *concepts*: “Every concept, as *partial concept*, is contained in the representation of things; as *ground of cognition*, i.e., as *mark*, these things are contained *under* it. In the former respect every concept has a *content* (*Inhalt*), in the other an *extension* (*Umfang*).” (*LL*, 593.) Kant states a principle of the inverse proportionality of extension and content: “The content and extension of a concept stand in inverse relation to one another. The more a concept contains *under* itself, namely, the less it contains *in* itself, and conversely.” (*LL*, 593.) In early works, Frege appeals to this principle to argue that a predicate which applied to all objects would have maximal extension, and so no content. (“Dialogue with Pünjer on Existence,” *NS*, 71/*PW*, 63; *GL* §29, 40; another use of the principle is in “Boole’s logical Calculus and the Concept-script,” *NS*, 16, fn **/*PW*, 15, fn **.)

Kant extended the distinction between content and extension from concepts to cognition in general, including judgments, speaking of the content of a cognition as a matter of its “richness,” “logical importance” and “fruitfulness” as the “ground of many and great

consequences,” and of the extension as a matter of the “horizon” of the cognition, the area within which it applied. (*LL*, 549-550.) In *BS*, Frege adapts this notion of content to the case of sentences; content is a matter of what is *implied* by or *contained* in a given claim.³

In thus adapting the notion of “content,” Frege was careful to mark a distinction, obscured in Kant’s talk of “judgment,” between the *act* of judging, and the *content* which is judged, calling the latter “judgeable content” (*beurtheilbarer Inhalt*) (*BS* §2, 1-2):

A judgment will always be expressed with the aid of the symbol

|—

which stands to the left of the symbol or combination of symbols giving the content of the judgment. If we omit the small vertical stroke at the left end of the horizontal one, then the judgment is to be transformed into a *mere combination of representations* (*blosse Vorstellungsverbindung*) of which the writer does not state whether he acknowledges its truth.

Shortly after *BS* Frege writes “Through this mode of notation I meant to have a very clear distinction between the act of judging and the formation of a mere judgeable content...” (“On the Aim of the ‘Conceptual Notation’,” 5.) This suggests that we “form” a judgeable content by combining “representations,” then judge by taking it to be true or false. While Frege addresses the distinction between content judged and act of judging, he is less clear here on the general “ing-ed” ambiguity, between represented content and act of representing, in the notion of “*representation*.” Thus in *BS* it is unclear what the precise status of the “representations” which are to be combined into judgeable contents might be – sometimes they seem to be representations in the sense of psychological acts, sometimes in the sense of the objects of those

acts.⁴

Frege's early account of conceptual content has often been taken to imply:

(A) any two "logical truths," or "analytic truths," have the same conceptual content.

Such a result would be clearly undesirable, given Frege's logicist thesis that the truths of arithmetic are analytic, derivable from the basic laws of logic together with definitions. Given **(A)**, his logicism would imply that the truths of arithmetic have the same content as the most trivial truths of logic, and that there is at most a psychological, but no logical difference, between judging that every natural number has a unique prime factorization and judging that the Moon is the Moon. Yet part of the point of Frege's logicism was to argue against Kant that analytic truths can extend our knowledge and so have genuine and distinctive content.

(A) is commonly taken to follow from another consequence of **(CONTENT)**:

(B) two sentences A and B have the same content just in case they mutually entail each other: $A \Rightarrow B$ and $B \Rightarrow A$.

(B) is often thought to be a direct consequence of **(CONTENT)**, but any *argument* from **(CONTENT)** to **(B)** must appeal to characteristics of the consequence relation \Rightarrow , and it is useful to make these explicit. One can argue that **(B)** follows from **(CONTENT)** using three general principles about consequence derivable from the natural deduction and sequent-calculus systems of logic devised by Gerhard Gentzen in 1935 (Gentzen, *Collected Papers*, 83-84):⁵

(1) *identity*: every sentence is a consequence of itself ($A \Rightarrow A$).

(2) *weakening*: superfluous premisses do not invalidate an inference. If a sentence A is a consequence of the set S ($S \Rightarrow A$), and T is a larger set than S ($S \subseteq T$), then A is a consequence of T ($T \Rightarrow A$).

(3) *cut*: if A is a consequence of S ($S \Rightarrow A$), and B is a consequence of S together with A ($S, A \Rightarrow B$), the “lemma” A may be “cut,” and B is a consequence of S alone ($S \Rightarrow B$).

From these principles, together with **(CONTENT)**, **(B)** follows.⁶ On the one hand, suppose that A and B have the same content. Then, as $A \Rightarrow A$ (identity), $B \Rightarrow A$ (by **(CONTENT)**, since B has all the consequences of A), and as $B \Rightarrow B$, also $A \Rightarrow B$ – that is, A and B are mutual consequences. On the other hand, suppose that A and B are mutual consequences, and suppose that $S, A \Rightarrow C$; then $S, A, B \Rightarrow C$ (weakening), and since $B \Rightarrow A$, we have that $S, B \Rightarrow A$ (weakening). Hence $S, B \Rightarrow C$ (cut). Similarly, if $S, B \Rightarrow C$, then $S, A \Rightarrow C$; so A and B have the same content (by **(CONTENT)**).

Frege does not provide an explicit theory of consequence in BS , so we can't be sure that he would have accepted all of Gentzen's principles. But, granting for the sake of argument that he would have, and so that he was implicitly committed to **(B)**, we cannot infer a commitment to **(A)** without further ado. *Given* a conception of logical truth as that which is a consequence of the empty set of premisses ($\emptyset \Rightarrow A$), we can argue from **(B)** to **(A)**: if A and B are logical truths, then $\emptyset \Rightarrow A$ and $\emptyset \Rightarrow B$, so $A \Rightarrow B$ and $B \Rightarrow A$ by weakening, and so they have the same content by principle **(B)**. However, it is doubtful that Frege would have accepted such an explication of logical truth. It is not clear what sense he could have made of a sentence being a “consequence of the empty set.” For Frege, consequence, following from, is a *relation* between judgeable contents which enables one judgment to be *justified* on the *basis* of *others*. He did not have Gentzen's notions of inference from an assumption and the discharging of assumptions, which could support a conception of a “proof” or “reasoning” without premisses. His systems of proof, both in BS and in the later *Grundgesetze* (GG), are devoid of rules such as conditional proof or *reductio ad absurdum* which rely on reasoning from assumptions. He represents apparent occurrences of

such reasoning in mathematical practice not as involving assumptions, but instead explicitly asserted hypothetical sentences with the seeming assumptions as antecedents.

Moreover, it is a consequence of this Gentzen-style conception of logical truth, together with Gentzen's principle of cut, that logical truths can always be "cut:"

(C) if $S, A \Rightarrow B$ and A is a logical truth, then $S \Rightarrow B$.⁷

It is doubtful that Frege would have accepted (C). One might argue for (C) as follows: logical laws collect and make explicit patterns of inference linking premisses and conclusions in valid reasoning. If we can infer B from premisses S together with logical law A , A 's only function must be to exhibit the link between the S and B . But this link must be there anyway, and so B must be a consequence of S without need of A . But this argument would imply that logical truths have no content and can in no way extend our knowledge, and this Frege must reject.

Frege might have accepted this argument for (C) in the case where the premisses in S and the conclusion B are *not* logical laws. But Frege's account of the argumentative structure of BS argues against his acceptance of (C) when S and B consist of logical laws. In BS , he sets out to prove various logical laws from a set of *basic* logical principles.⁸ He writes (BS §13, 25.):

It seems natural to deduce the more complex of these judgments from the simpler ones – not to make them more certain, which would in general be unnecessary, but to bring out the relations of the judgments to one another. Merely knowing the laws is obviously not the same as also understanding how some are implicitly contained in others. In this way we obtain a small number of laws in which (if we add the laws contained in the rules) is contained, though in embryonic form, the content of all of them. And it is an advantage of the deductive mode of presentation that it teaches us to recognize this kernel. Because

we cannot enumerate all of the boundless number of laws that can be established, we can attain completeness only by a search for those which, *potentially*, imply all the others. This passage makes little sense if any logical law follows from any other. For Frege, clearly, not all logical laws are on a deductive par.

In an unpublished paper, “Boole’s logical Calculus and the Concept-Script,” Frege explains his choice of axioms for *BS*: he only “assumed such as appeared necessary for the proof” of the final proposition (133) of the book. He adds: “that my sentences have enough content ... follows from the fact that they were adequate to the task” – that they sufficed for the proof of (133). (*NS*, 43/*PW*, 38.) Had he omitted some of his axioms, and been unable to carry out the proof, the resulting collection of axioms would *not* have had “enough” content.

Frege comments that in carrying out his proofs, he had, at times, to employ “diminution[s] in content” through the addition of “superfluous conditions” as “necessary transition points.” (*NS*, 43/*PW*, 38.) He has in mind cases like this: aiming to prove C , one proves $(A \supset B) \supset C$ and B ; one then “diminishes” the content of B by using an instance of axiom (1) of *BS*, $B \supset (A \supset B)$, to add the “superfluous condition” A , arriving at $A \supset B$, from which one can infer C .⁹ Even if both B and $A \supset B$ are logical laws, for Frege the latter has “diminished” content compared to the former. Not everything which follows from B also follows from $A \supset B$; notably, B itself does not. In a similar vein, Frege says that he “had to assume formulae which merely express the different ways in which you may alter the order of a number of conditions. Instead of giving a general rule that conditions may be ordered at random, I only introduced a much weaker axiom that two conditions may be interchanged, and then derived from this the permissibility of other transpositions.” (*NS*, 43/*PW*, 38.) He has in mind here axiom (8), $(d \supset (b$

$\supset a)) \supset (b \supset (d \supset a))$), from which he derived a series of theorems, such as (12), $(d \supset (c \supset (b \supset a))) \supset (d \supset (b \supset (c \supset a)))$), about which he comments “Propositions (12)-(17) and (22) show how, when several conditions are present, their order can be altered.” (*BS* §16, 37.) It would have been simpler to introduce a general rule to this effect, as in his later *Grundgesetze*,¹⁰ but that would have been to choose a stronger principle than necessary, *given* the other axioms of *BS*. Given those axioms, (8) is sufficient, although from (8) alone one cannot prove (12) – axioms (1) and (2) are needed as well.

For Frege there is an ordering of the logical laws themselves, which the system of *BS* lays bare. Frege would not admit that the first axiom of *BS*, proposition (1), and its last theorem, proposition (133), are mutual consequences. (133) does not follow from (1) alone; the proof of (133) requires other axioms of *BS* as well. Such a proof reveals the logical interconnections of the propositions of *BS*, which Frege records in a table at the end of the work, indicating which propositions are used in the proofs of subsequent propositions. “Consequence” is already for Frege a notion with epistemological import. Deducing consequences from basic logical laws is a process which generates content, insofar as the conclusions we deduce are contained in the basic laws collectively, but not individually. Later, when he was sure that he had proved that the laws of arithmetic are analytic truths, deducible from the basic laws of logic and definitions, Frege writes: “Can it be said that the sentence ‘ $3+7 = 10$ ’ is deduced from the sentence ‘ $2^2 = 4$ ’? Hardly. Is ‘ $3+7 = 10$ ’ a consequence of ‘ $2^2 = 4$ ’? Apparently not...” (“On Mr. Peano’s Conceptual Notation and My Own” (1897), 372.) Here again, he rejects the view that every logical or analytic truth is a consequence of every other. Thus, the supposed difficulty for his *BS* conception of content, posed by principle **(B)** is void.¹¹ For although Frege might plausibly be

held to accept principle **(B)**, he clearly would have rejected the sorts of additional principles necessary to move from **(B)** to **(A)**.

None of this, however, is to say that he had a worked out theory of the consequence relation. Rather, he relied on a working appreciation of the ways in which we count sentences as following from, or not following from, others. As a working mathematician, Frege knew that it was inappropriate to treat proposition (133) of *BS* as following from proposition (1) – if he had offered as “proof” of (133) the “reasoning,” “(1), therefore (133),” he would have failed in his goal of establishing that “pure thought ... is able, all by itself, to produce from the content which arises from its own nature judgments which at first glance seem to be possible only on the grounds of some intuition” (*BS* §23, 55) or, as he put it in 1884, commenting on the proof of (133), “sentences which extend our knowledge can contain analytic judgments.” (*GL* §91, 104.)

Nonetheless, Frege’s practical sense of what counts as a consequence of what, and so of which sentences count as the same in content, presents serious difficulties in connection with providing any general account of the notions of logical law and logical consequence, and so, also, of conceptual content. It is natural to suppose that the logical laws are the sentences provable in *BS* (or some suitably expanded version of it), and that *B* is a consequence of *A* if and only if the conditional $A \supset B$ is a logical law. This would imply, however, that any two logical laws are consequences of one another, and so have the same content. Some more refined notion of logical consequence is needed, perhaps agreeing with that given here when neither *A* nor *B* are logical laws. But, in *BS*, Frege provided no such account. Still, he seems to have been aware that there was a problem here. When he claimed that his “sentences have enough content” because they are “adequate to the task” of proof, he added a *caveat*: “in so far as you can talk of the

content of sentences of pure logic at all.” (“Boole’s logical Calculus and the Concept-Script,” *NS*, 43/*PW*, 38.) This hesitancy concerning talk of the content of “sentences of pure logic” was overcome by his desire to show that *content* – “richness,” “logical importance” and “fruitfulness” as the “ground of many and great consequences” – could arise from these sentences.¹²

BS’s notion of judgeable content is one root of the sense-meaning distinction; more precisely, it is the ancestor of the later notion of the *thought*, the *sense* of a sentence. Much later in his career, in 1906, he returned to the issues of the proper characterization of consequence, and so also of the individuation of content, or as he now put it, the individuation of thoughts.

Frege’s attempt to characterize consequence occurs at the end of his controversy with Hilbert over the foundations of geometry. Having rejected Hilbert’s approach to proving the independence of the axioms of geometry, he tries to give a proper account of independence, and so also “dependence” (“On the Foundations of Geometry: Second Series, III,” 423-4):

Let Ω be a group of true thoughts. Let a thought G follow from one or several of the thoughts in this group by means of a logical inference such that apart from the laws of logic, no sentence not belonging to Ω is used. Let us now form a new group of thoughts by adding this thought G to the group Ω . Call what we have just performed a logical step. Now if through such a sequence of steps, where every step takes the result of the preceding one as its basis, we can reach a group of thoughts that contains the thought A , then we call A dependent upon group Ω .

Frege limits the scope of this account, however (“On the Foundations of Geometry,” 425):

In taking a logical step from the thought-group Ω , we are applying a logical law. The

latter is not to be counted among the premises and therefore need not occur in Ω . Thus there are certain thoughts, namely the laws of logic, which are not to be considered when dealing with questions concerning the dependence of a thought.

Here, he seems aware that the proposed account of consequence, if applied to logical laws, would make logical laws dependent upon one another, and takes pains to block this result.

In the same year, Frege twice took up the question of the individuation of thoughts, once in a letter to Husserl of 9 December, 1906, and once in an unpublished manuscript, “A brief Survey of my logical Doctrines.” The explanations he provides are similar in interesting ways, but also differ in important respects. He writes to Husserl (*WB*, 105-106 /*PMC*, 70-71):

It seems to me that an objective criterion is necessary for recognizing a thought again as the same, for without it logical analysis is impossible. Now it seems to me that the only possible means of deciding whether sentence *A* expresses the same thought as sentence *B* is the following, and here I assume that neither of the sentences contains a logically self-evident component part in its sense. If *both* the assumption that the content of *A* is false and that of *B* true *and* the assumption that the content of *A* is true and that of *B* false lead to a logical contradiction, and if this can be established without knowing whether the content of *A* or *B* is true or false, and without requiring other than purely logical laws for this purpose, then nothing can belong to the content of *A* as far as it is capable of being judged true or false, which does not also belong to the content of *B*; for there would be no reason at all for any surplus in the content of *B*, and according to the presupposition above, such a surplus would not be logically self-evident either. In the same way, given our supposition, nothing can belong to the content of *B*, insofar as it is capable of being

judged true or false, except what also belongs to the content of *A*. Thus what is capable of being judged true or false in the contents of *A* and *B* is identical, and this alone is of concern to logic, and this is what I call the thought expressed by both *A* and *B*.

In “A brief Survey,” on the other hand, he explains (*NS*, 213-4/*PW*, 197):

Now two sentences *A* and *B* can stand in such a relation that anyone who recognizes the content of *A* as true must thereby also recognize the content of *B* as true and, conversely, that anyone who accepts the content of *B* must straightaway (*ohne weiteres*) accept that of *A*. (*Equipollence*). It is here being assumed that there is no difficulty in grasping the content of *A* and *B*. ... I assume that there is nothing in the content of either of the two equipollent sentences *A* and *B* that would have to be immediately accepted as true by anyone who had grasped it properly...

So one has to separate off from the content of a sentence that part that alone can be accepted as true or rejected as false. I call this part the thought expressed by the sentence. It is the same in equipollent sentences of the kind given above. It is only with this part of the content that logic is concerned.

Both these explanations share with the *BS* account of conceptual content a concern to isolate that *part* of a sentence’s content which is “of concern to logic.” Further, both explanations attempt to isolate this part through a sort of mutual consequence test – *BS*’s account ultimately reduces to such a test, assuming Gentzen’s principles governing consequence. In the letter to Husserl, Frege spells this out in terms of a sort of *reductio ad absurdum* procedure, whereas in “A brief Survey” his approach is more direct: anyone who accepts *A* ought also to accept *B* (reading “must” (*müssen*) here as having *normative* force). The most significant

difference between the two accounts, though, comes with the word(s) “straightaway” (*ohne weiteres*) in “A brief Survey” – this suggests that in order to express the same thought, *A* and *B* must be mutual *immediate* consequences, whereas the procedure outlined in the letter to Husserl would count as equipollent sentences which are not *obviously* mutual consequences.

But most significant of all is the fact that each account explicitly omits from consideration sentences containing “a logically self-evident component part,” or something “that would have to be immediately accepted as true by anyone who had grasped it properly.” Frege again seems aware that without such a restriction, his explanations would entail that any two logical laws express the same thought, and takes steps to ward off this conclusion. Thus, in his attempts to characterize logical consequence, and to individuate thoughts, Frege in 1906 displays a concern with issues that troubled us in our discussion of his 1879 account of judgeable content.

II. *Begriffsschrift*: subsentential content

Frege’s *BS* account of conceptual content led him to reject the traditional distinction between subject and predicate as irrelevant to content. His initial example of sentences with the same content showed that the same thing can appear as subject, or as predicate, without changing the content. Frege replaced this traditional analysis of judgments with an analysis based on the mathematical notions of function and argument. In the Preface to *BS*, he says that “it is easy to see how regarding a content as a function of an argument leads to the formation of concepts.” (*BS*, xiii.) However, in *BS* itself, the notions of function and argument are explained for linguistic expressions, rather than contents. If we start with a complex expression, with or without a judgeable content, we can view a part of this expression as replaceable by other expressions. This yields an analysis of the whole expression into a part which is held constant –

the function – and a part which is left to vary – the argument. (*BS* §9, 15-16.) Two points are crucial about this explanation. First, there is more than one way to analyze a complex expression into function and argument. For example, the sentence “Cato killed Cato” can be analyzed into the argument “Cato” and the functions “() killed Cato,” “Cato killed (),” or “() killed ()” (among others). Second, functions, unlike the expressions from which they are formed, are “incomplete” – they have argument-places which need to be filled to form an expression with a complete content.

In papers written shortly after *BS*, as well as in the *Grundlagen der Arithmetik (GL)* of 1884, Frege extends this function-argument analysis to support an account of the formation of *concepts*. His plan is simply to transfer the replacement and omission model of *BS* from the *expressions* of judgeable contents to the contents themselves. Thus, beginning with the content *Cato killed Cato*, we form the concept of suicide by viewing the *content* of the word “Cato” as replaceable in both its “occurrences.”¹³ This plan requires that judgeable contents be constructed in a manner analogous to the sentences that express them, so that we can speak of the “occurrences” of the contents of sub-sentential parts of sentences in the judgeable contents that the sentences express; and it requires a conception of the content of sub-sentential parts of sentences. All of this is governed by a principle of compositionality: the content of a complex expression is composed out of the contents of the parts of that expression, in a manner analogous to the way in which the expression is composed out of its parts. This compositionality of content has two consequences: first, the content of a part of an expression is a part of the content of the whole expression; and second, if two expressions have the same content, substituting one of these expressions for the other within a larger complex expression will not change the content of

the whole.

The simplest form of sub-sentential expression is a proper name, such as “Cato” in our example. What is the content of such a name? The answer implicit in Frege’s account of concept-formation is: the object named by the name, in this case Cato himself.¹⁴ The objects about which we judge are built into the judgeable contents themselves. Frege’s model of judgeable content thereby simultaneously accounts for two kinds of norms governing our cognitive and linguistic acts of judging and asserting – on the one hand norms of logical consistency, inferential interconnection, and generally responsibility of judgments to each other, and on the other hand, norms of truth, and generally responsibility to the world. Judgeable contents are individuated in terms of their consequences; hence, when one judges a given content to be true, it is determined what further contents one is committed to judge true as well. But the content which one judges true also contains as parts the objects about which one judges, and which determine whether the correctness of one’s judgment. The concepts employed in judging are factored out of judgeable content by omitting the objects which figure in it. Concepts’ dual character of content and extension derives from this factoring. A concept stands derivatively in consequence relations to other concepts, determined by the relations of the contents obtained by “completing” them, and so can be said to have as content all concepts which “follow” from it. At the same time, a concept determines a class of objects to which it applies, the class of objects which complete the concept to form a correct judgeable content; these make up the extension of the concept.

However, all is not well with this model. There seems to be an instability in trying to combine in one “content” the two normative dimensions of truth, responsibility of judgment to

the world, and inference, responsibility of judgments to one another. By building the objects about which we speak and think directly into judgeable contents, we risk identifying contents that we want to distinguish on inferential grounds. Frege is not unaware of this difficulty in *BS*, where he attends to a special case of it, involving the concept of identity, a primitive logical sign in *BS*.

The problem arises as follows. Suppose that $a=b$, that is, a and b are the very same thing, and consider the sentences “ $a=a$ ” and “ $a=b$.”¹⁵ These sentences have different consequences, and so must differ in content. From “ $a=b$,” together with “ Fa ,” we can infer “ Fb .” From “ $a=a$,” no such conclusion follows. Yet, since $a=b$, the names “ a ” and “ b ” have the same content. Thus substituting one for the other should not change the content of the whole. But such a substitution transforms “ $a=b$ ” into “ $a=a$.” So these sentences must have the same content.

Frege responds to this dilemma in *BS* by proposing a meta-linguistic account of the identity sign – in identity contexts, names stand not for their contents but for themselves. (*BS* §8, 13.) Thus, “ $a=b$ ” says that “ a ” and “ b ” have the same content. This affords a way out of the difficulty. We do not have to conclude that if $a=b$, “ $a=b$ ” and “ $a=a$ ” have the same content. *In this context*, even though $a=b$, “ a ” and “ b ” do not have the same content, as long as they are distinct names – since *in this context* the content of “ a ” and “ b ” is the names themselves.

Frege recognizes an objection to this solution, however. On this view, assertions of identity “pertain to the *expression* and *not to the thought*,” concerning only our form of expression, and not the things of which we would speak. (*BS* §8, 14.) This leads easily to the conclusion that in a perspicuous *Begriffsschrift*, we have no need for different names for one thing, or for a sign of identity. If the job of a name is to stand for its content, and we adhere to

the principle, one sign for one job, the sign of identity will be superfluous.

Frege uses a geometrical example to respond to this objection. Beginning with a point *A* lying on a circle, another point is constructed as the point of intersection of a certain line and the circle. One then discovers that the first point and the second point are one and then same. (*BS* §8, 14.) Frege goes on to explain that in this example, “the same point is determined in two ways:” “directly in intuition,” and “as the point *B* corresponding to” the given construction.

Frege’s argument here echoes another distinction from Kant’s *Logic*: that between the matter and form of a cognition.¹⁶ Kant says that the matter of a cognition is the object, whereas the form is “*the way in which* we cognize the object.” He too provides an example (*LL*, 544-545):

If a savage sees a house from a distance... with whose use he is not acquainted, he... has before him in his representation the very same object as someone else who is acquainted with it determinately as a dwelling established for men. But as to form, this cognition of one and the same object is different in the two. With the one it is *mere intuition*, with the other it is *intuition* and *concept* at the same time.

Frege’s example, like Kant’s, involves an object being given both intuitively and conceptually. However, Frege’s talk of different ways of “determining” the same point modifies Kant’s formulation in one key respect: *distinct* objects cannot be *determined* in the *same* way. In Kant’s terms, where the form is the same, so must be the matter.

Frege uses his example to dispel the impression that the identity sign is dispensable: “That the *same content*... is ... given by *two modes of determination* is the content of a *judgment*.” (*BS* §8, 14.) The identity sign allows us to express such contents. When distinct

names “*a*” and “*b*” are associated with distinct modes of determining the same object, the identity sentence “ $a=b$ ” will bring this out. In such cases, Frege asserts, distinct names for one object, far from being superfluous, “concern the very heart of the matter” (*das Wesen der Sache selbst betreffen*), and identity judgments are “in Kant’s sense, synthetic” – they extend our knowledge through having new and useful consequences. (*BS* §8, 15.) Frege thus officially introduces the identity sign in the following way: “ $a=b$ ” asserts that the “*a*” and “*b*” have the same content, so that each can be substituted for the other wherever it occurs. (*BS* §8, 15.) Thus the inferential content of “ $a=b$ ” is preserved – from it, together with the sentence “*Fa*” we can infer the sentence “*Fb*,” and indeed one of the basic axioms of *BS*, “ $a=b \supset (Fa \supset Fb)$,” codifies this inference.

There is, however, an initial difficulty with this way of explaining the function of “ $=$.” Frege first asserted that “ $a=b$ ” states that the names “*a*” and “*b*” have the same content; later he claimed that “ $a=b$ ” expresses the judgment that “the same object is determined in different ways.” This seems to provide two ways of understanding the content of “ $a=b$ ”: (1) as a claim about the *names* “*a*” and “*b*,” that they have the same content; (2) as a claim about the associated *modes of determination*, that they determine the same thing. It is not obvious that these are compatible. We will see that Frege’s introduction of the sense-meaning distinction is intertwined with an attempt to address this problem.

In *BS*, Frege applies his identity sign to sentences, expressions of judgeable contents. He introduced his notion of conceptual content through a natural-language example of distinct sentences with the same content. The apparatus of *BS* generates similar cases, involving sentences which Frege recognizes as mutually inferrable, since their equivalence is “obvious”

enough that we can count each as following directly from the other. The simplest such case is that of double negation – the distinct sentences “ A ” and “ $\sim\sim A$ ” express the same content.¹⁷ Two of the axioms of *BS* are “ $\sim\sim A \supset A$ ” (proposition 31) and “ $A \supset \sim\sim A$ ” (41). (*BS* §§18-19, 44, 47.) In the Preface, however, Frege says that these axioms “can be combined into the single formula $\vdash (\sim\sim A = A)$.” (*BS*, viii.) This new axiom states that “ $\sim\sim A$ ” and “ A ” have the *same content*, according to the *BS* account of “ $=$.” One might argue that all Frege is really claiming is the *material* equivalence of “ A ” and “ $\sim\sim A$,” since this is all that is stated by (31) and (41). However, the proposed new axiom is *not* a mere replacement for (31) and (41) but a real enrichment of *BS*, which contains no principles from which one could derive “ $\sim\sim A = A$ ”. There is no possibility in *BS* of deducing “ $A = B$ ” from the material conditionals “ $A \supset B$ ” and “ $B \supset A$.”¹⁸

It has been frequently pointed out that the *BS* theory of identity is beset with problems.¹⁹ For example, the theory implies that in the *BS* axiom “ $a=b \supset (Fa \supset Fb)$,” “ a ” and “ b ” are ambiguous, standing for themselves in “ $a=b$,” and for a and b in “ Fa ” and “ Fb .” This makes conceptual trouble for the use of this axiom when “ Fa ” is replaced by an identity context, an application which Frege needs to deduce the symmetry of identity (at proposition (55)).²⁰ One such application yields “ $a=b \supset ((a=a)=(a=b))$ ” – in other words, if “ a ” and “ b ” have the same content, so do “ $a=a$ ” and “ $a=b$.” This demonstrates the bankruptcy of the proposed solution in the context of the formal system of *BS* – the formal system itself treats identity as a relation between the objects named, not between names.

Avoiding these difficulties while retaining the *BS* account of “ $=$ ” would require a major overhaul of the formal system of *BS*. But the *BS* account of identity fails at a more fundamental level, for it does not really address the problem it was intended to solve. That problem was that

in treating the objects about which we judge as parts of the contents which we recognize as true, we end up conflating contents that we want to hold apart because they do not have the same consequences. The *BS* account of identity addresses this issue only in the case of identity sentences, by allowing that even when $a=b$, “ $a=a$ ” and “ $a=b$ ” can have different content. But the general *BS* account of content still implies that when $a=b$, “ Fa ” and “ Fb ” have the same content, and so are mutually inferable. This, however, violates our intuitions about what follows from what. We do not suppose that, merely because $a=b$, one who asserts “ Fa ” and denies “ Fb ” thereby contradicts herself, as would happen if “ Fb ” were a consequence of “ Fa .” Generalizing the *BS* account of identity to handle all such problems would lead to the unfortunate result that in all contexts, names stood for themselves rather than for their content.

These sorts of difficulties were very much in Frege’s mind at the time of the composition of “On Sense and Meaning” (“*Über Sinn und Bedeutung*,” *S&B*, 1892) and *Grundgesetze der Arithmetik (GG I, 1893)*. However, in the years following the publication of *BS*, Frege continued to work with the picture of content we have sketched here. In his “logicist manifesto,” *Grundlagen der Arithmetik (GL)* of 1884, the *BS* picture is assumed throughout, and plays an important role in his account of the Kantian analytic-synthetic distinction, with which he frames the project of the book.²¹ However, by the time of the publication of his second great logicist work, *GG*, the basic picture of content adumbrated so far had been replaced by the famous theory of sense and meaning. In my view the sources of this fundamental reworking of Frege’s thought are to be found in the development of his logicist project in *GL*.

III. Objectivity, Objecthood and the Context Principle in *Grundlagen*

Frege wrote *BS* with the aim of establishing the epistemological status of arithmetic. In

Part III of that work, he proved some results in a “general theory of sequences,” which he hoped to be able to apply to the sequence of natural numbers. Five years later, in *GL*, he developed the philosophical basis of his claim that, *contra* Kant, arithmetic is analytic, and that its truths can be proved from logical laws and definitions alone. In *GL*, he claimed only to have made this plausible informally; the formal demonstration, making use of the apparatus of *BS*, was reserved for *GG*, whose first volume appeared nine years after *GL*. In *GL* the need for developing a more careful account of judgment and content began to reveal itself; and the project of *GG* led to the full-fledged theory of sense and meaning.

Frege prepares the ground for his own account in *GL* with a critique of other philosophies of arithmetic. In the course of his argument, he emphasizes both the *objectivity* of mathematical truths, and the *objecthood* of the numbers which mathematics studies. He thus sets himself against two tendencies in the philosophy of mathematics, against which he polemicized throughout his subsequent career: psychologism and formalism. The former takes mathematical terms to mean *ideas*, while the latter avoids ascribing any meaning to them whatsoever.

Frege’s argument turns on “three fundamental principles” (*GL*, x):

always to separate sharply the psychological from the logical, the subjective from the objective

never to ask for the meaning of a word in isolation, but only in the context of a sentence

never to lose sight of the distinction between concept and object.

The first principle rejects psychologism; the third principle, Frege says, implies that “a widely held formalist theory ... is untenable.”²² The second of the three principles, the “context

principle,” is the lynchpin on which the others depend.²³ Frege claims that if the context principle is violated, “one is almost forced to take as the meanings of words mental pictures in the individual mind, and so to offend against the first principle as well.” He is less explicit about the relation between the context principle and the concept-object distinction, but it is there nonetheless. In *GL*, Frege gets at the distinction between concept and object through a distinction between *names* and *concept-words* (predicates), itself drawn with the help of the context principle. It is only by considering how a word functions in a sentence that we can determine its logical place as name or predicate, and so determine the place of its content as concept or object.

Frege’s distinction between the psychological/subjective and the logical/objective leads him to reconsider the Kantian vocabulary of “representation” (*Vorstellung*) and “content” employed in *BS*. He now recognizes the ing-ed ambiguity of “representation.” “in compliance with the first principle, I have used the word ‘representation’ always in the psychological sense, and have distinguished representations from concepts and from objects.”²⁴ (*GL*, x.) In a footnote to a discussion of the view that “number is the representation of the position of an item in a series” he writes (*GL* §27, 37):

My arguments would be beside the point if he meant by representation an objective idea (*Idee*); but in that case what difference would there be between the representation of the position and the position itself?

A representation in the subjective sense is what is governed by the psychological laws of association; it is of a sensible, pictorial character. A representation in the objective sense belongs to logic and is in principle non-sensible, although the word which

means an objective representation is often accompanied by a subjective representation, which nevertheless is not its meaning. Subjective representations are often demonstrably different in different men, objective representations are the same for all. Objective representations can be divided into objects and concepts. I shall myself, to avoid confusion, use “representation” only in the subjective sense.

Here, Frege distinguishes between the *act* of representing, the “subjective representation,” and the content represented, the “objective representation.”²⁵ In *BS* Frege marked the distinction between the content of judgment and the act of judging in his logical notation, through his “judgment-stroke.” However, he was not yet completely clear on the subjective-objective distinction drawn in *GL*, as is shown by his treatment of judgeable content as a “mere combination of representations,” “formed” by a mental act. *GL*, in contrast, emphasizes the objectivity of content.

If arithmetic is an objective science, it must have its “objective representations,” objects and concepts. Frege insists on a sharp distinction between objects and concepts, and argues that numbers are not concepts, but objects. The notion of concept here is modeled on the linguistic idea of “function” of *BS*. That account of function, however, presupposes a category of “complete” expressions which are not functions, and which serve both as the “wholes” within which we can omit and vary parts to obtain functions, and as the “parts” which we omit and vary.²⁶ Similarly, the account of concepts in *GL* presupposes both judgeable contents and objects as parts of those contents. Yet the context principle might seem to undercut the needed distinction by making all subsentential expressions equally “incomplete,” and all non-judgeable contents equally dependent on judgeable contents.

To resolve this difficulty we must attend to Frege's actual use of the context principle in *GL*. When he writes "Only in a sentence have the words really a meaning. ... It is enough if the sentence taken as a whole has a sense; it is this that confers on its parts also their content" (*GL* §60, 71), it is tempting to take this in one of two extreme ways: (1) the content of a word is determined by the sense of *any* sentence in which it occurs; (2) the content of a word is determined by the sense of *all* sentences in which it occurs. Either of these possibilities would make it difficult to sustain a real distinction between concept and object. Neither of these suggestions, however, reflects Frege's intention.

Frege approaches the nature of numbers through the context principle: "It should throw some light on the matter to consider number in the context of a judgment which brings out its basic use." (*GL* §46, 59.) He considers "statements of number" such as "there are two senators from Indiana" and concludes that "a statement of number contains an assertion about a concept" (we can say of the concept *senator from Indiana* that two individuals fall under it). This might lead to the view that numbers are "second-order concepts" (the number two is the concept under which such concepts as *senator from Indiana* and *prime less than 5* fall). However, Frege denies that numbers are such concepts. Rather, they are "self-subsistent objects that can be recognized as the same again." (*GL* §56, 68.) Frege distinguishes concepts and objects by the *kind* of questions one can ask about them, the *kind* of sentences in which they occur. "With a concept, the question is always whether anything, and if so what, falls under it. With a proper name, such questions make no sense." (*GL* §51, 64.) On the other hand, the "self-subsistence," and thus objecthood, of numbers "comes out at every turn, as for example in the identity $1 + 1 = 2$." (*GL* §57, 69.)

Frege's use of the context principle shows that contexts of the form "*a* is *F*" are primary in establishing the content of a predicate like "*F*", whereas contexts of the form "*a=b*" are primary in establishing the content of a name like "*a*". The "self-subsistence" of objects *consists* in their being "recognizable as the same again," as expressed in statements of identity. Thus Frege can claim: (*GL* §60, 72.)

The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a sentence, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning.

This interpretation of Frege's use of the context principle is borne out by his answer to the Kantian question: "How then are numbers given to us, if we cannot have any representations or intuitions of them?" He appeals to the context principle: "Since it is only in the context of a sentence that words have any meaning, our problem becomes this: to define the sense of a sentence in which a number word occurs." But he immediately narrows this problem: "we have already settled that number words are to be understood as standing for self-subsistent objects. And that is enough to give us a class of sentences which must have a sense, namely those which express our recognition of a number as the same again." The problem thus becomes "to construct the content of a judgment which can be taken as an identity such that each side of it is a number." (*GL* §§62-3, 73-74.)

However, the use of *identity* contexts as the crucial marker of namehood, and so objecthood, puts considerable pressure on the *BS* account of identity. For it requires *both* that identity be a relation between objects, rather than between names, *and* that non-trivial identity

statements be possible. If identity were only a relation between names, the use of names in identity contexts could hardly “confer content” on them in such a way as to “give” us objects. But equally, if identity statements were always trivial, like “ $a=a$,” an ability to form such sentences and judge them to be true would have no interesting consequences. Yet the *BS* account preserved the non-trivial character of identities only by treating identity as a relation between names and not objects.

In *GL* these difficulties surface in Frege’s attempts to define the numbers. His stated aim to “construct the content of a judgment which can be taken as an identity such that each side of it is a number” shows that he understands identity as a relation between objects which are parts of the content of an identity judgment.²⁷ On the other hand, he emphasizes the importance of non-trivial identities, arguing that if we assumed that every object could be given in only one way,

All identities would then amount simply to this, that whatever is given to us in the same way is to be reckoned as the same. This, however, is a principle so obvious and so sterile as not to be worth stating. We could not, in fact, draw from it any conclusion which was not the same as one of our premisses. Why is it, after all, that we are able to make use of identities with such significant results in such diverse fields? Surely it is rather because we are able to recognize something as the same again even although it is given in a different way.²⁸ (*GL* §67, 78-79)

This passage clearly harks back to the *BS* account of content and identity, pointing out that identities often have non-vacuous *content*, serving as the ground for novel consequences. At this point however, Frege had no account of how non-trivial identities are possible, compatible with a treatment of identity as a relation between objects. Such an account would have wait for the

theory of sense and meaning.

IV. “On Sense and Meaning:” Introduction

We come now to the official presentation of Frege’s views in his most famous essay, “On Sense and Meaning” (*S&B*, 1892). Frege begins *S&B* with essentially the picture of “content” sketched in sections I and II above. But he now recognizes the instability of this picture – the contents assigned to names on the one hand and to sentences on the other cannot fit together in the way demanded by the principles of substitution and compositionality. The meta-linguistic account of identity in *BS* was an attempt to get around this problem. *S&B* opens with a reconsideration of this account.

Frege begins by asking whether identity is a relation, and if so whether it relates objects or names.²⁹ He states that in *BS* he had taken to relate names, and says that he will expound the reasons for this view. The rest of the long opening paragraph of *S&B* is often thought to contain not only the argument for the earlier view but also some or other argument against it; but a careful comparison of *BS* and *S&B* shows that Frege is, just as he says, simply recapitulating the argument for the *BS* account. We will follow this paragraph closely.³⁰ (*S&B*, 25-6.)

First, Frege outlines the problem posed by identity statements. If identity relates objects, it seems that “ $a=b$ ” and “ $a=a$ ” must have the same content, since they state the same relation to hold between the same objects. This argument turns on the idea that name-content is part of sentence-content – the two identity sentences here are composed out of the same parts in the same way. However, Frege points out, the two sentences have manifestly different “cognitive value” (*Erkenntniswert*). “ $a=a$ ” is a trivial consequence of the law of identity – it has no interesting consequences. “ $a=b$ ” may require investigation to discover; it can be a fruitful

discovery, rich in consequences. “The discovery that the rising sun is not new every morning, but always the same, was one of the most fertile astronomical discoveries. Even today the re-identification of a small planet or comet is not always a matter of course.” (*S&B*, 25-6.) Clearly, these sentences differ in content, in their *consequences*. In later writings, Frege sometimes uses the terminology of “content” explicitly to make this point. For example, in correspondence with Peano, he writes that “anyone can see that the thought of second sentence [‘The evening star is the same as the morning star’] is different, and in particular that it is *essentially richer in content* (*wesentlich inhaltreicher*) than that of the former [‘The evening star is the same as the evening star’].” (Frege to Peano, undated, *WB*, 196/*PMC*, 127.)

The *BS* theory was meant to provide for this worry. It assigns different content to the two sentences by treating “ $a=b$ ” as saying that the signs or names “ a ” and “ b ” stand for the same thing. However, Frege remarks, “this is arbitrary.” He thus raises the worry expressed in *BS*, that the meta-linguistic theory rescues the wrong *sort* of content for identity-statements, content which “pertains merely to the *expression* and *not to the thought*.” (*BS*, 14.) “The sentence $a=b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means.”³¹ (*S&B*, 26.)

In *S&B*, Frege clarifies this worry by asking what our talk of “names” or “signs” here amounts to. He asks whether “the sign ‘ a ’ is distinguished from the sign ‘ b ’ only *as an object* (here, by means of its shape)” or “*as a sign* (*i.e.* ... by the manner in which designates something).” (*S&B*, 26, my emphasis.) When we speak of “signs,” we might mean “mere” signs, natural objects which we may put to some specific linguistic use, but which are not individuated by any features of that linguistic use. In that case, as Frege puts it, the use to which a sign is put

is arbitrary – we could equally put that sign to some other use or put some other sign to that use. But we might also mean signs-in-use, individuated “as signs,” not *only* by such features as shape or size, but *also* by the linguistic use to which they are put.

Frege had already implicitly deployed such a distinction in his critique of formalism in *GL*, where he wrote that “an empty sign (*Zeichen*) ... without some content ... is merely ink or print on paper, as which it possesses physical properties,” but “really ... would not be a sign at all.” (*GL* §95, 107.) Shortly after *GL*, Frege made an explicit distinction between “figure” (“*Figur*”) and “sign” (“*Zeichen*”), the former possessing “geometrical, physical and chemical properties,” whereas the latter essentially has the “purpose of designating.” (“On Formal Theories of Arithmetic” (1885), 97-8.) In his critique of formalism, Frege emphasized the difference between an *empty* figure to which no content is assigned, and a true *sign*, complete with content. But in later writings, he also applied this sort of distinction in the case of what could be called figures-in-use. For example, in “Compound Thoughts” (1918-19) he writes (39):

As a mere thing, of course, the group of letters ‘and’ is no more unsaturated than any other thing. It may be called unsaturated in respect of its employment as a sign (*Zeichen*) meant to express a sense, for here it can have its intended sense only when situated between two sentences: its purpose as a sign requires completion by a preceding and succeeding sentence.

Similarly, he emphasizes that a sentence (*Satz*) is a “group of signs that expresses a thought,” so that if a different thought were associated with the same group of sign-designs, “it would not even be the same sentence.” (“On the Foundations of Geometry, Second Series, II,” 377; “On the Foundations of Geometry: First Series, I,” 323.) More generally we can say that the same figure

can be used in different ways, yielding different signs; we have the *same* sign if and only if we have the same figure, put to the same use.³²

Adapting a suggestion of Wilfrid Sellars (“Abstract Entities”) to this context, let us use “asterisk quotes” to form names of the “figures” enclosed in them; and let us reserve ordinary quotation for naming “signs,” figures-in-use. So, for example, *and* occurs twice in *Sand and water make mud*, while “and” occurs only once in “Sand and water make mud.” Now suppose that “ $a=b$ ” says that the *figures* * a * and * b * are used to stand for the same thing. Here “used” must mean something like “used by me/us.” We could then rewrite “ $a=b$ ” as something like

$$\exists x(U(i, *a*, x) \& U(i, *b*, x))$$

where “ $U(x,y,z)$ ” says that x uses y as a sign for z – there is something which I use both * a * and * b * to stand for. This expresses no “proper knowledge” about the objects a and b , but only linguistic knowledge about the use to which I put the figures * a * and * b *. Why? Consider what *follows* from the identity statement “ $a=b$ ” so understood. The answer is – nothing, except more statements about the use of figures. In particular, from “ $a=b$ ” so understood, together with the premiss “ Fa ”, the conclusion “ Fb ” does *not* follow. For there are possible worlds in which I use both the expressions *Augustus* and *Julius* to refer to Tiberius, while in those worlds Augustus, and not Julius, remains the first emperor.³³ It is not quite true that on the *BS* view of identity so understood, there is *no* difference in content between “ $a=a$ ” and “ $a=b$ ” – there is a difference, but it concerns only “our mode of expression,” not the objects a and b themselves. But this amounts, for present purposes, to no difference worth speaking of. This is why Frege says that “if the sign ‘ a ’ is distinguished from the sign ‘ b ’ only as an object... not as a sign..., the cognitive value of $a=a$ becomes essentially equal to that of $a=b$, provided $a=b$ is true.”

The moral of the argument so far is not to refute the *BS* account but to show that it has been misconceived. What we need to do is to view “ $a=b$ ” not as specifying a relation between mere figures, but between signs, individuated “as signs,” thus, in part, by the linguistic use to which they are put. The question is *exactly* how to understand this. We had better not understand the “use” of a name purely in terms of standing for a particular object. If we do that, we will treat the two signs “ a ” and “ b ” as essentially the same, differing only by an arbitrary choice of figure, when “ $a=b$ ” is true, and we will be left with precisely the view that “ $a=b$ ” never expresses any proper knowledge. As the two signs would differ only as figures, the only knowledge we would express by ‘ $a=b$ ’ would be the knowledge that the figures **a** and **b** have been put to the same use. So we have to individuate names not only through their use as standing for an object, but through some other feature of their use as well. Frege now reiterates the same reworking of the Kantian form-matter distinction as that given in *BS*: a sign is to be individuated by the “*manner* in which it designates something,” the “mode of presentation” (*Art des Gegebenseins*, mode of being given) of the thing designated.”³⁴ As in *BS* he provides a geometrical example, involving two “different designations for the same point,” which “likewise indicate the mode of presentation.” Hence “the statement [of identity] contains actual knowledge.” (*S&B*, 26.)

We can now view the *BS* account of identity in this way: “ $a=b$ ” is to be explained as

$$\exists x(D("a", x) \& D("b", x))$$

where “ $D(x,y)$ ” says that *sign* x designates y . “ a ” and “ b ” are here individuated *as signs*, (in part) by their associated modes of presentation, and so we need not include any mention of the user. Moreover, Frege assumes that we cannot have two objects given in the same way, by the same “mode of presentation.” It follows that if we were to use the figure **Julius** to refer to Tiberius,

we would have a different mode of presentation, and hence a different *sign*. We literally could not use the *sign* “Julius” to refer to Tiberius. Thus the relation of *sign* to object is no longer a matter of arbitrary choice. Hence it is possible to express proper knowledge through an identity statement understood in this way. Yet we are still working with the *BS* account; all we have done is to carefully spell out that account so as to avoid some obvious difficulties with it.

Having explained the reasons lying behind the *BS* view of identity, Frege immediately moves into a discussion of the sense-meaning distinction, to which we will turn in a moment. But it is noteworthy that his opening paragraph leaves some big questions dangling. Is identity, after all, a relation between names? And how *exactly* do we account for the specific cognitive value of identity statements (not merely the possibility of their expressing real knowledge, but the specific content which makes them into useful discoveries)? Frege returns to these issues only very briefly at the end of the essay and even then they are not entirely resolved. The explanation of this must wait on a full-fledged spelling out of the advances in Frege’s conception of content, of the act of judgment, and so of knowledge itself. These advances come on two fronts: the treatment of the content of names, and the treatment of the content of sentences. Together they are designed to provide us with a unified picture which both builds on and replaces the original *BS* account.

V: “On Sense and Meaning”: names

Frege follows up his discussion of the *BS* theory of identity by introducing the terminology of sense and meaning. He says that we can associate with each name, in addition to the object named, its “meaning,” a “sense” in which “the mode of presentation is contained.” (*S&B*, 26.) This move reflects the *BS* suggestion that the content of a judgment of identity is

“that the same content ... is actually given by two modes of determination.” (*BS* §8, 14.) That idea, when pressed, yielded the view that the parts of the content of “ $a=b$ ” are *not* the names “ a ” and “ b ,” but the modes of determination with which they are associated. “ $a=b$ ” turns out to be *about* these modes of determination; it is they rather than the objects named which are what the names “ a ” and “ b ” stand for in this context. *This* thought is not retained in the mature picture. What is retained is the idea that it is the modes of determination/presentation of the objects named which are the parts of the content, or *sense*, of the whole sentence.

In introducing the sense-meaning distinction for names, Frege began with the *BS* model in which the content of a name is the object that it names, while the content of a sentence is individuated in terms of its consequences. This model is in tension with the principles of compositionality and substitution, as is revealed by the problem of identity. The *BS* theory of identity resolved this tension through a special reinterpretation of the identity sign. Careful examination of this theory has now shown that there is a semantically significant aspect of names over and above the “content” or object named, which is not reducible to the arbitrary choice of a particular figure to name that object. This feature Frege identifies as the mode of presentation of the object. Since the “content” of names, as object named, turns out to be inappropriate to serve as a constituent of the “content” of sentences as individuated by consequences, Frege searches for a feature of names which can be more suitably pressed into that role. Within the confines of the basic *BS* picture, nothing *could* be pressed into that role other than the mode of determination/presentation of the object named. This is a reflection of the fact that, for the Frege of *BS*, names refer “directly” to objects, although they may do so in different ways.³⁵

Frege never goes beyond the metaphor of “mode of presentation” to provide a more

detailed theory of sense. He does, however, make several remarks which clarify the role of the notion of name-sense in his account of language.³⁶ The sense of a name, he tells us, is grasped by anyone sufficiently familiar with the language to which the name belongs. Ideally, to each sign in a given language there will correspond exactly one sense, and to each sense exactly one meaning. However, in natural language, ambiguity is possible – the same word can have different senses in different contexts.³⁷ Similarly, it is possible to form expressions which possess a sense but no meaning, for example “the least rapidly converging series.” (*S&B*, 27-8.) Frege claims that in a proper scientific *Begriffsschrift*, every proper name must have a meaning; in *GG* he attempts to prove this of his own system of notation.³⁸ (*GG I*, xii; §§28-31, 45-50.) It is therefore a “fault” of language to permit the formation of names with sense but no meaning, just as it is a fault of language to allow ambiguity.³⁹ (*S&B*, 40.)

Frege emphasizes that the sense of a name is expressible in different languages, and is objective. Thus, it is distinguished from the idea or representation (*Vorstellung*) called up by the name. The latter is subjective, private, and psychological, the former is objective and communicable. He provides an image to elucidate this distinction (*S&B*, 30):

Somebody observes the Moon through a telescope. I compare the Moon itself to the meaning; it is the object of the observation, mediated by the real image projected by the lens (*objective Glas*) in the interior of the telescope, and by the retinal image of the observer. The former I compare to the sense, the latter is like the representation or intuition (*Vorstellung oder Anschauung*). The optical image in the telescope is indeed one-sided and dependent on the standpoint of observation; but it is still objective, inasmuch as it can be shared by several observers.... But each one would have his own

retinal image.

This analogy is meant to highlight the shareability and objectivity of the notion of sense, but it has another important feature. The “real image” in the telescope is not a “third thing” intervening between the observer and the star which she sees “through” it. Similarly, sense as “mode of presentation” need not be seen as a “third thing” intervening between speaker and meaning. The idea that the “way in which the object is given” is a semantically significant feature of a name need not detract from the idea that objects are given to us “directly.”⁴⁰ Thus worries, such as troubled Russell, that the notion of sense leads to skeptical doubts like those generated by representationalist theories of knowledge of classical British empiricism, may be misplaced.⁴¹ Yet this way of thinking about sense is in tension with Frege’s repeated commitment to the possibility of sense without meaning. If sense is simply the way in which an object is given to us, and not an entity intervening between us and the object, the possibility of sense without meaning becomes mysterious. The interpretation of Frege developed by Gareth Evans and John McDowell takes off from this observation. According to Evans and McDowell, Frege’s “better self” would reject the possibility of sense without meaning. While there is much to this line of thought, the full force of this issue can only be appreciated after we have developed the other side of Frege’s sense-meaning theory, his account of the sense and meaning of sentences. As we will see, the possibility of sense without meaning plays a crucial role in Frege’s arguments concerning the meaning of sentences, yet sits poorly with the resulting account of judgment and truth.

VI: “On Sense and Meaning”: sentences

After elucidating the sense-meaning distinction for names, Frege turns to the “content” of

declarative sentences, which we use in assertion to express acts of judgment.⁴² He begins by stating that a sentence contains (*enthält*) a “thought” (*Gedanke*), by which he means “not the subjective performance of thinking but its objective content (*Inhalt*), which is capable of being the common property of many thinkers.” (*S&B*, 32, fn 5.) I take this to be a reference to his earlier doctrine of “judgeable content,” individuated in terms of consequences.⁴³ Frege then argues for the following claims (*S&B*, 32-34):⁴⁴

- (1) the thought expressed is the sense of a sentence, not its meaning.
- (2) a sentence must in some cases have a meaning as well as a sense.
- (3) the meaning of a sentence is a “truth-value.”
- (4) truth-values are objects, and sentences, the names of those objects.

His arguments show that he has decided in advance that the principle of substitution must apply to both sense and meaning, insofar as these notions can be applied to complex linguistic expressions. His procedure follows the pattern exhibited for names – starting with the picture of names as standing for objects and sentences as expressing thoughts, Frege sought something “on the level of thoughts” associated with names, and now he seeks something “on the level of objects” associated with sentences, in each case guided in part by the principle of substitution. But his argument also shows the special character of sentences as those expressions through which we express acts of judgment, therefore acts at least potentially of knowledge; this dimension of sentences plays a dominant role in the argument especially for (2)-(3). This belies the common charge that Frege made a decisive mistake in rejecting the fundamental categorial distinction between sentences and names, judgeable and unjudgeable contents, central to his earlier thought.⁴⁵

Frege argues for (1) in two steps. First, he argues (1a) that the thought expressed by a sentence is not its meaning; second, he concludes immediately (1b) that the thought is the sense. This conclusion reflects a determination to make do with the two categories of sense and meaning. This should not be surprising – name-sense was introduced as something which could serve as the name-counterpart of sentential content/thought. We see here a fundamental reorganization of the *BS* picture: the *BS* view that the object named, the “content” of the name, is part of the thought expressed, the “content” of the sentence, is replaced by the parallel, yet fundamentally different view that the sense of the name (the mode of presentation of the object named) is part of the sense of the sentence (the thought expressed).

Frege’s argument for (1a) turns on the substitution principle:

- (i) there are cases in which names “*a*” and “*b*” have the same meaning, but sentences “*Fa*” and “*Fb*” do not express the same thought
- (ii) if “*a*” and “*b*” have the same meaning, and “*Fa*” and “*Fb*” have a meaning, “*Fa*” and “*Fb*” have the same meaning (by the substitution principle)
- (iii) therefore, the thought expressed is not the meaning of the sentence which expresses it.

The crucial step in this argument is (i). This claim fits well with our discussion of the problem of identity – if substitutions of names with the same meaning always preserved the thought expressed, there would be no more content to “*a=b*” than to “*a=a*”, since the supposed additional content of allowing such inferences as that from “*Fa*” to “*Fb*”, would reduce to a matter of the trivial verbal reformulation of the same thought. Frege argues to the contrary that “*Fa*” and “*Fb*” in such cases need not express the same thought because one who did not know that *a=b* might hold the one to be true and the other to be false.

This might seem to be a simple application of the indiscernibility of identicals, but such an argument would fail. One who holds “*Fa*” to be true and “*Fb*” false might also hold “*Fb*” true and “*Fa*” false, albeit unwittingly, if these express the same thought. What is crucial here is the claim that one who did not know that $a=b$ might hold “*Fa*” true while *failing* to hold “*Fb*” true. Frege puts it this way in a letter to Russell (Frege to Russell, 21 May, 1903, *WB*, 240/*PMC*, 157-158):

Now the thoughts contained in these sentences are evidently different; for after having recognized the first as true, we still need a special act to recognize the second as true. If we had the same thought there would be no need for two acts of recognition, but only for a single one.

Thus “*Fa*” and “*Fb*” do not express the same thought because one who holds “*Fa*” to be true need not immediately, and without a special cognitive act, hold “*Fb*” true as well.⁴⁶

Having established (1) that the thought expressed by a sentence is its sense, not its meaning, Frege goes on to argue (2) that at least some sentences have a meaning as well as a sense. His argument for (2) can be summarized roughly as follows.⁴⁷

(i) there are contexts, such as fiction and poetry, in which we are not interested in the meanings of our words, but only their sense; in such contexts we require only that our sentences express a thought, not that they have a meaning.

(ii) we are led to ask after the meanings of our words only in the context of the search for *truth*.

(iii) therefore, there must be something associated with each sentence, which (a) depends on the meanings of the words making up the sentence, and (b) accounts for our interest in the meanings of words in the context of the search for truth.

(iv) this feature of a sentence can be called its meaning, since it depends on the meanings of the words making up the sentence.

The contrast Frege draws between poetry and fiction, on the one hand, and the search for truth, on the other, allows him to claim that there is something distinctive about the second case, which requires the introduction of meaning for sentences as well as names. Frege claims that in contexts of fiction and poetry we care only about the thoughts expressed by our sentences. He argues that these thoughts do not depend on the meanings of our words, so that even if those words were to lack meaning altogether, our sentences would still express thoughts, so long as our words had a sense. For example, the thought expressed by the sentence “Odysseus was set ashore at Ithaca while sound asleep” “remains the same whether ‘Odysseus’ means something or not.” (*S&B*, 33.) If we were to discover that Odysseus in fact existed, this would not change the thoughts expressed by the sentences in the *Odyssey* one bit. However, it would put us in a position to ask whether the sentences in the story were true – a question which we would not even care to raise so long as we were just taking the story *as* a story, as fiction or poetry.

This question of truth requires that our words have meaning as well as sense, according to Frege. And it is this question which drives him to conclude that at least some *sentences* must have a meaning as well as a sense. He concludes almost immediately (3) that this meaning must be the “truth-value” of the sentence, *true* or *false*, since it is only when we are inquiring after the truth-value that we are led to take an interest in meanings.⁴⁸ Finally, he asserts (4) that the two truth-values are *objects*, “*the True*” and “*the False*,” of which sentences are *proper names*, since sentences, like names and unlike predicates and other functional expressions, are “complete.”

This way of speaking may sound artificial, as Frege recognized, and is often taken to be a

serious retrograde step, going back on *BS*'s recognition of the crucial difference between sentences and names, judgeable and unjudgeable contents. However, Frege is engaged here in a fundamental rethinking of his account of judgment, and when this is appreciated it can be seen that in fact Frege retains a basic distinction between sentences and names insofar as it is only names of truth-values which can be used to make assertions, and so to express judgments.

VII: “On Sense and Meaning”: Judgment and Identity⁴⁹

Judgment, Frege tells us, can be regarded as the “advance from a thought to a truth-value.” (*S&B*, 35.) This is to be contrasted *not* with the earlier formula that judgment is the recognition of the truth of a content, which Frege repeats numerous times in his later writings with “thought” replacing “content,” but with the earlier conception of the *grammar* of that formula. The earlier formula seems to imply that truth is a *property* of contents or thoughts, and that judgment is a special case of the more general act of recognizing something as *F* – the special case of recognizing something as true. In the “Logic” of the 1880’s, in which Frege introduces his account of judgment as the recognition of truth, he writes of the importance of “the *property (Eigenschaft) ‘true’*” for logic. (*NS*, 4/*PW*, 4, my emphasis.) But in *S&B* Frege tells us that the relation of thought to truth-value is not that of subject to predicate, but rather that of sense to meaning. (*S&B*, 34.) Thus, the act of recognizing something as *F*, where *F* is an ordinary property of things, is a special case of judging, of recognizing a thought as true, rather than the other way around. Hence we cannot conceive of judging as recognizing that a thing (thought) has a property (truth). Rather judgment is “something quite peculiar and incomparable,” which we get at metaphorically as an “advance from a thought to a truth-value.” (*S&B*, 35.) To say that this is an advance from sense to meaning, not from a thought to one of its

properties, is a way of gesturing at the unique status of judgment, and so also of truth. To say that *the True* is an object is another way of gesturing at the unique status of truth by denying that it is a concept, the meaning of an ordinary predicate. In the later “Logic” of 1897, Frege avoids talk of a property of truth, writing instead that “the *word (Wort)* ‘true’ can be used to indicate ... a goal for logic” and emphasizing the “peculiarity” of this grammatical predicate. (*NS*, 139-140/*PW*, 128-9, my emphasis.)

In support of the claim that truth is not related to thought as subject to predicate, Frege points out that the ostensible subject-predicate sentence “the thought that *p* is true” in fact says nothing more than “*p*”. (*S&B*, 34.) These two sentences, used assertorically, make exactly the same truth-claim; on the other hand, when uttered without assertoric force, neither makes a truth-claim at all. Hence, the essential truth-claim is not made by using the predicate “true” but rather in the act of asserting. Frege concludes that “the relation of the thought to the True may not be compared with that of subject and predicate,” since “subject and predicate ... are just elements of thought; they stand on the same level for knowledge. By combining subject and predicate one reaches only a thought, never passes from sense to meaning, never from a thought to its truth-value.” (*S&B*, 35.) The point of all this is to rethink both the nature of truth and of judgment.

If judgment is an advance from thought to truth-value, it is also an advance from sense to meaning; but subsentential expressions such as names, as well as sentences, are said to have sense and meaning. We can now see that Frege has achieved a *reorganization* of the basic *BS* picture while retaining essentially the same *elements*. The reorganization comes through the placing of what were essentially “name-content” and “sentence-content” on different levels, the levels of meaning and sense; but other aspects of the original picture are picked up as correlates

of these so that both names and sentences have both sense and meaning. What makes it reasonable to say that we have two *levels*, that truth-value is to sentence as object named is to name, or that mode-of-presentation is to name as thought is to sentence, is that the principle of substitution holds at both levels. This principle unifies the “levels” and make it reasonable to use one term (“sense,” “meaning”) across each level. We have seen the principle of substitution for meaning at work in Frege’s arguments that thoughts cannot be the meanings of sentences. Frege later explicitly adopted the principle of compositionality for senses, and from this the principle of substitution for senses follows.⁵⁰

In *S&B*, Frege seems to commit himself to the principle of compositionality for meaning as well, suggesting that “judgments are distinctions of parts within truth-values.” (*S&B*, 35.) Frege later apparently contradicts this, noting that the meaning of a part of an expression need not be a part of the meaning of the whole expression – while “Sweden” is a part of “the capital of Sweden,” Sweden is not a part of Stockholm, for example. (“Notes for Ludwig Darmstaedter” (1919), *NS*, 275/*PW*, 255.) However, careful examination of his suggestion in *S&B* shows that already there he has hedged his bets so as to avoid this objection. He first says that the “distinction of parts” within a truth-value “occurs by a return to the thought” so that there will be a “mode of analysis” of the True (the False) for each true (false) thought. He notes however that he has “used the word ‘part’ in a special sense,” differing from its ordinary use. Normally, given the whole and the part we would be able to determine a unique remainder, but given a truth-value and an object we cannot uniquely determine a concept – “2 is prime” and “2 is even” are both true. Hence “a special term would need to be invented” for this notion of part-hood. (*S&B*, 36.)

In Frege's later writings he does not continue to speak of word-meanings as parts of truth-values; but the deepest point of the metaphor, and so of the idea of compositionality for meanings, is retained. This is the point that in judgment, the recognition of a thought as true, we are not directed merely to a thought and a truth-value, but also to the meanings of the parts of the sentence, the objects which the sentence is about and the concepts which are applied to those objects.⁵¹ These are what he metaphorically calls the "parts" of the truth-value. Judgment is that act in which we are directed to truth, and it is *thereby* that act in which we are led to ask for the meanings of our words. Judgment requires that we grasp thoughts as *articulated* in a way that directs us to objects and concepts. In Frege's later writings he makes the same point by insisting that our sentences are *about* the objects and concepts which are the meanings of the words we use. Only if our sentences are about objects and concepts can we express truths through them.

In later writings ("Logic" (1897), *NS*, 139-40/*PW*, 128-9; "Thoughts" (1918-19), 59-60) Frege builds on his argument that truth is not a property, to show that truth is indefinable.⁵² He argues as follows. If truth were definable, we would have available some such definition as:

the thought that p is true = φ (the thought that p)

where φ is some perhaps complex defining phrase, for example

the thought that p is true = the thought that p corresponds to a fact.

To use such a definition in order to determine whether the thought that p was true, we would have to judge whether or not φ (the thought that p) – we would need to recognize *this* as true, or reject it as false. This would require yet another application of the definition, yet another judgment, and so on, resulting in an infinite regress. Consequently, the purported definition cannot be used, and so is no definition at all; and truth has to be recognized as indefinable.

This has one important consequence which Frege does not explicitly draw, however. Just as truth is not a property of thoughts, so there is no “relation of meaning” between senses and meanings.⁵³ For if there were such a relation, truth would be a *definable property* of thoughts:

the thought that *p* is true = the thought that *p* means the True.

But if there is no *relation* of meaning between senses and meanings, to say that the relation of thought to truth-value is that of sense to meaning rather than of subject to predicate is at best to say something metaphorical and elucidatory of judgment and truth, both of which are indefinable, simple, and quite “peculiar and incomparable.”⁵⁴ This highlights the fundamental place of judgment in all our talk of sense and meaning. It is only in the context of judgment that the “relation” of sense to meaning is approached, not only for sentences, but for the words that make them up. It is only in the context of judgment, the striving for truth, that we become interested in the *meanings* of our words. Here there is an echo of the context principle of *GL*⁵⁵ – it is only in the act of grasping a complete thought and judging as to its truth that we take “the step from the level of thoughts to the level of meaning (the objective).” (*S&B*, 34.) While Frege had earlier emphasized the objectivity of judgeable content, the point here is that it is only in judging that we move to the objective level of truth-evaluability, the level at which our thought makes contact with the meanings of our words. The echo of the context principle here shows that Frege has not simply abandoned his insight into the difference between sentences and names, judgeable and unjudgeable contents, but has relocated it in the context of a richer theory of the act of judgment.

In *S&B*, Frege bases his argument on the claim that “*p*” and “the thought that *p* is true” say the same thing. In other writings, he extends this claim in two ways, each of which might

seem problematic. In the unpublished “Logic” of 1897, he claims that “ p ” and “it is true that p ” express the same thought (*NS*, 153/*PW*, 141; also Frege to Russell, 13 November, 1904, *WB*, 245/*PMC*, 163); and in “Logic in Mathematics” (1914) he comes close to claiming that “ p ” and “ p is true” are similarly equivalent. We discuss these in turn.

Frege’s claim that “ p ” and “it is true that p ” say the same thing might seem to be in conflict with his account of “indirect discourse” contexts such as “Jones believes that p .” In such contexts, according to Frege, words do not stand for their ordinary meanings, but rather for their ordinary senses, which are their “indirect meanings.” (*S&B*, 28.) Thus in “Jones believes that Smith is a fool” the name “Smith” stands for its ordinary sense, and the subordinate sentence “Smith is a fool” stands for the thought that Smith is a fool. One might suppose that it is the business of the word “that” to effect this transformation, so that “ p ” in any clause of the form “that p ” stands for a thought.⁵⁶ In that case, we would have to conclude that “ p ” in “it is true that p ” stands for a thought, and it would be hard to see what work “it is true that” could play other than to predicate truth of the thought.

Yet a careful consideration of Frege’s long discussion of such cases in the second half of *S&B* should dispel this worry. Frege nowhere says that “that” (or even “the thought that”) automatically generates an indirect discourse context. Rather, he attends to the specific behavior of each context in which a subordinate clause is involved. Of particular interest is his treatment of contexts of the form “ S fancies that p .” He claims that in such contexts, we have the simultaneous expression of two thoughts: “ S believes that p ” and “not- p ”. He comments (*S&B*, 48):

In the expression of the first thought, the words of the subordinate clause have their

indirect meaning, while the same words have their customary meaning in the expression of the second thought. This shows that the subordinate clause in our original complex sentence is to be taken twice over, with different meanings: once for a thought, once for a truth-value.

Here the little word “that” does not prevent the words in the subordinate clause from having their customary meaning (albeit in addition to their indirect meaning).

Frege’s reasoning here is guided by two principles. First, if we cannot substitute words with the same customary meaning within a larger sentential context, *salva veritate*, this shows that the words have their indirect meaning; but if we *can* perform such substitutions then we should take the words to have their customary meaning – this is the *default* position which failures of substitution upset; the doctrine of indirect meaning is introduced only to account for such failures. Second, if we find that a truth-functional context involving “*p*” (such as “not-*p*”) is a *consequence* of a context “ $\varphi(\text{that } p)$ ” we should take it that the words occurring in this context have their customary meanings. This principle reflects the ancestry of his notion of thought in his earlier consequence-driven conception of content. The two principles together account for his diagnosis of “*S* fancies that *p*.” But they also imply that in the context “it is true that *p*” words have their customary meanings, and no other. We can always substitute words with the same customary meaning in this context *salva veritate*, so we have no grounds for taking the words to have other than their customary meaning; and, since “it is true that *p*” has “*p*” as a consequence, we have a reason to take the words to have their customary meaning. Even in the context “the thought that *p* is true” we do not have a reference to a thought. This sentence says no more than “*p*.”

In “Logic and Mathematics,” Frege assimilates the case of “‘ p ’ is true” to that of “the thought that p is true:” “to say of a sentence, or thought, that it is true is really quite different from saying of sea water, for example, that it is salt. In the latter case we add something essential by the predicate, in the former we do not.” He concludes that this shows “that truth is not a property of sentences or thoughts,” and “confirms that a thought is related to its truth value as the sense of a sign is its meaning.” (*NS*, 252/*PW*, 234.) This discussion strongly suggests that, like “the thought that p is true,” “‘ p ’ is true” is equivalent in sense to p .

Yet it may seem that Frege is wrong to assimilate “the thought that p is true” and “‘ p ’ is true” in this way. For he holds that truth is primarily ascribed to thoughts and only derivatively to the sentences that express them. (“Thoughts,” 60.) This suggests the following schema, using “ \approx ” to indicate sameness of sense:

the thought that p is true $\approx p$

“‘ p ’ is true \approx “‘ p ’ expresses the thought that p and the thought that p is true

\approx “‘ p ’ expresses the thought that p and p .

Here it seems that “‘ p ’ is true” has content over and above “‘ p ,” shown in the clause “‘ p ’ expresses the thought that p .”

However, bearing in mind the figure-sign distinction drawn above, we must ask whether by “‘ p ” we are to understand a figure or a sign. In the first case, the above schema becomes:

* p * is true \approx * p * expresses the thought that p and the thought that p is true

\approx * p * expresses the thought that p and p .

Here there is indeed additional content in the clause “* p * expresses the thought that p ,” since the use of a particular complex figure to express a particular thought is a matter of arbitrary choice.

Indeed, there is an intelligible sense in which a *property* of truth for propositional *figures* has been explained here, although all the *interesting* work is done by the *expression* relation between figures and thoughts. But in the second case (directly represented by the first schema above, given our decision to use quotation to name signs, not figures) the supposed additional content can be discounted; for the sentential *sign* “*p*” expresses the thought that *p essentially*. Frege states in “Thoughts” that he uses the word “sentence” in *this* sense, so that “the sense necessarily goes with the sentence.” (“Thoughts,” 68; see also “Foundations of Geometry, First Series, I,” 323.) Hence there is no conflict between his claim that truth is ascribed primarily to thoughts and derivatively to sentences, and his claim that “‘*p*’ is true” and “*p*” express the same thought.

I claimed above that, as a consequence of Frege’s thesis that truth is not a property of thoughts, there is no real relation between sense and meaning. However, if this is so, what can we make of such claims as that name “*a*” means object *b*, or that the sense of “*a*” means *b*? I suggest that we take our cue from Frege’s argument that truth is not a property, *since* “‘*p*’ is true,” “the thought that *p* is true” and “*p*” say the same thing. In order to simplify the following exposition, let us adapt another of Sellars’s devices, and use “dot-quotation” to abbreviate “the sense of the name ‘___,’” so that “•Caesar•” abbreviates “the sense of the name ‘Caesar’.” Now consider “*a*” and “the meaning of •*a*•.” It is plausible that these expressions have not only the same meaning, namely *a*, but also present this meaning in the same way, that is in the way that “*a*” presents *a*. Thus, we can view these two expressions as possessing not only the same meaning but also the same sense. Furthermore, if we assume that the name “*a*” is individuated *as a sign* and not as a figure, “*a*” will have its sense essentially, and so we can conclude that all three expressions “*a*,” “the meaning of ‘*a*,’” and “the meaning of •*a*•” express the same sense.

Finally, since “‘ a ’ means b ” can be rephrased as “the meaning of ‘ a ’ = b ,” this expresses the very same thought as “ $a=b$.”

Frege at one point (“Comments on Sense and Meaning,” 1892-95) makes a similar suggestion in connection with his well-known difficulties with talking about concepts. He holds that “the concept Φ ” refers to an object, not a concept, because it is a complete expression, and lacks the predicative nature of a concept-word. He notes that “the meaning of the concept-word A ” is in the same boat, and concludes (*NS*, 133/*PW*, 122.):

Indeed, we should really outlaw the expression ‘the meaning of the concept-word A ’, because the definite article before ‘meaning’ points to an object and belies the predicative nature of a concept. It would be better to confine ourselves to saying ‘what the concept word A means’, for this at any rate is to be used predicatively: ‘Jesus is, what the concept word “man” means’ *in the sense of (in dem Sinne von)* ‘Jesus is a man’.”

What is noteworthy for our purposes here is the suggestion that “is a man” and “is, what the concept word ‘man’ means” are equivalent not only in meaning but in *sense*.

We can now return to our worries about Frege’s opening discussion of identity in *S&B*. At the end of that discussion, Frege left the initial question whether identity is a relation between names or between objects unresolved. Moreover, he gave no detailed account of the specific cognitive value of identity statements, which played such a central role in his argument.

Our account of the so-called “meaning-relation” helps with the first of these worries. Frege had asked whether identity related objects or names, citing the *BS* as holding the latter view. We saw that the formal system of *BS* implicitly treats identity as relating objects, and that the project of *GL* similarly requires that identity relate objects, not merely names. Frege’s later

explanations of the identity sign sometimes seem to confirm that this is his view. For example, in *GG* he introduces his identity sign in this way: “‘ $\Gamma = \Delta$ ’ shall denote the True if Γ is the same as Δ ; in all other cases it shall denote the False.” (*GG I* §7, 11.) In “Comments on Sense and Meaning,” he states that “the relation of equality, by which I understand complete coincidence, identity, can only be thought of as holding for objects, not concepts.” (*NS*, 130-1/*PW*, 120.) Yet in his later writings Frege also makes statements that seem to fit better with the *BS* view. Earlier in *GG*, for example, he says that “if I wrote ‘ $(2 + 3 = 5) = (2 = 2)$ ’ ... I should only have designated the truth-value of ‘ $2 + 3 = 5$ ’s denoting the same as ‘ $2 = 2$ ’.” (*GG I* §5, 9.) And in “Comments on Sense and Meaning” he treats “The meaning of the word ‘conic section’ is the same as that of the concept-word ‘curve of the second degree’” and “The concept *conic section* coincides with the concept *curve of the second degree*” as interchangeable. (*NS*, 131/*PW*, 120.)

One might be disposed to dismiss such statements as hand-waving pedagogy in the context of informal exposition. But this thought is undercut by a striking example in which Frege seems to assert the *BS* view *in the midst* of discussing the importance of distinguishing use and mention. Frege opens “Function and Concept” (1891), the first published work in which he deploys the sense-meaning distinction, by criticizing views which confuse “form and content, sign and thing signified.” He sees such confusion as underlying the view that “ $2+5$ and $3+4$ are equal but not the same” – “difference of sign” is transferred to “difference of thing signified.” Against this, he says: “What is expressed in the equation ‘ $2 \cdot 2^3 + 2 = 18$ ’ is that the right-hand complex of signs has the same meaning as the left-hand one.” (“Function and Concept,” 3.)

In *S&B* the opening question as to whether identity is a relation between names or between objects is never actually answered. The apparent conflict is now easily resolved,

however. The sentences “ $a=b$,” “the meaning of ‘ a ’ = the meaning of ‘ b ’”, and “the meaning of $a=b$ = the meaning of $a=b$ ” not only have the same truth-value, they express the same thought. Moreover, in spite of linguistic appearances, none of these sentences states a relation between anything but the objects a and b , just as none of the sentences “ p ”, “ p is true” and “the thought that p is true” expresses anything but p .

We can now return to the worry raised by Frege’s initial introduction of sense as a response to the problem of identity. The problem was to explain how the two sentences “ $a=a$ ” and “ $a=b$ ” can differ in content, “cognitive value,” while expressing the same relation between the same things. The basic answer is that while the two names “ a ” and “ b ” have the same meaning, they may differ in sense, by presenting the same object in different ways. Frege explains at the end of *S&B*: “If we found ‘ $a=a$ ’ and ‘ $a=b$ ’ to have different cognitive values, the explanation is that for the cognitive value (*Erkenntniswert*) the sense of the sentence, viz., the thought expressed by it, is no less relevant than its meaning, i.e. its truth-value (*Wahrheitswert*).”⁵⁷ When the sense of “ a ” differs from that of “ b ”, “the thought expressed in ‘ $a=b$ ’ differs from that of ‘ $a=a$ ’... the two sentences do not have the same cognitive value. If we understand by ‘judgment’ the advance from the thought to its truth-value... the judgments are different.” (*S&B*, 50.)

To fully appreciate Frege’s point here, we must recognize the intimate connections between *judgment*, *inference*, and *knowledge* in his thought.⁵⁸ To judge is to recognize (*anerkennen*) the truth of a thought. The German term “*anerkennen*,” like its English translation, points to a link between judgment and knowledge.⁵⁹ Judgment is essentially oriented towards truth: the truth of the thought that is recognized to be true is internal to the act of judgment, as its

goal. But truth is not the only goal of judgment. Judgment aims, finally, at *knowledge*, and for this truth is not enough. For the purposes of knowledge, “the thought expressed ... is no less relevant than [the] truth-value.” “Cognitive value” (*Erkenntniswert*, value for knowledge) and “truth-value” (*Wahrheitswert*) are both species of “value” in the sense of a goal or end.⁶⁰

Inference, for Frege, is a process whereby we make judgments on the basis of other judgments; there is no such thing as inference from a mere assumption or hypothesis. The role of inference is to justify some of our judgments on the basis of others. To say that one thought has another as a consequence, then, is to say that anyone who recognizes the first to be true would be in a position to justifiably recognize the second to be true, by an inference from the first. Thus inference, like judgment, is intrinsically ordered to truth and knowledge. This will imply, as a necessary condition, that if one thought has another a consequence, it is impossible for the first to be true and the second false; but it is implausible to attribute to Frege the idea that this is also a sufficient condition. For one thought to have another as its consequence, the drawing of the inference from the first to the second must *justify* the recognition of the truth of the second. But as we have seen, though for any two logical laws, it is impossible that the first be true and the second false, nonetheless some logical laws are *basic* and others not; such basic laws will not be *consequences* of non-basic laws on this account.

Now, if judgment is the advance from a thought to a truth-value, when we advance from different thoughts to the same truth-value, our acts of judgment are different, and so the knowledge acquired is different. Thus we can see how the difference in sense between “*a*” and “*b*” can translate into a difference in “cognitive value” between “*a=a*” and “*a=b*.” Moreover, we can begin to see how to understand the *specific* difference in cognitive value, the difference in

content, in consequences, between “ $a=a$ ” and “ $a=b$.” The question is, fundamentally, why given “ $a=b$ ” we can infer from “ Fa ” to “ Fb ,” whereas given “ $a=a$ ” we cannot.

Suppose that one has recognized the truth of “ $a=b$ ”. One has thereby grasped a thought which has as parts two senses, $\bullet a\bullet$, the sense of “ a ,” and $\bullet b\bullet$, and recognized the truth of that thought. Now suppose further that one has judged that Fa . Here one has grasped another thought, which has as a part $\bullet a\bullet$, and recognized its truth. Further, suppose that one has also grasped the thought that Fb , which has as a part $\bullet b\bullet$. One can see here at least the beginnings of an explanation of how it is possible that all this should amount to a justification for the further recognition of the truth of the thought that Fb . For after all the two thoughts whose truth one has recognized are related in a familiar way to the third yet to be recognized as true, the first premise containing $\bullet a\bullet$, the second linking this to $\bullet b\bullet$, and the conclusion replacing $\bullet a\bullet$ with $\bullet b\bullet$ in the first. However, this purely formal explanation might seem insufficient; for after all the same could be said about any so-called inference of the form “ Fa ”, “ aRb ”, therefore “ Fb ”.

It is tempting to try to supplement this explanation in one of two ways. First, one might try to argue as follows. I have claimed that “ $a=b$ ” and “ $\bullet a\bullet$ has the same meaning as $\bullet b\bullet$ ” express the same thought. Consequently in judging that $a=b$ we have judged that $\bullet a\bullet$ and $\bullet b\bullet$ have the same meaning, and it is this specific link that enables us to replace the one sense with the other and infer from the judgment that Fa to the judgment that Fb . But clearly this “explanation” is circular; it employs exactly the principle that it is intended to explain. Second, one might suggest that since $\bullet a\bullet$ and $\bullet b\bullet$ give us the very same object, when we have grasped both senses properly we will be enabled to infer from the judgment that Fa to the judgment that Fb . But this move eliminates the need for the premise “ $a=b$ ” and ends up again reducing the content of “ $a=b$ ” to

that of “ $a=a$ ”. For Frege it is crucial that the same object can be given to us in different ways without our *knowing* that this is so, in other words without our knowing that $a=b$.

The proper response to this situation is to give up the search for an *explanation* of the difference in inferential potential between “ $a=a$ ” and “ $a=b$.” The theory of sense and meaning is misconceived if it is thought of as providing an explanatory account of the facts concerning inferences and identity. More generally, we should refuse to assign explanatory priority to any of the various concepts of truth, judgment, inference, thought, object, mode of presentation, naming and so on. This does not mean that the theory is left without a point or a purpose, however. For the theory displays in its categories – name, sentence, sense, meaning, thought, truth-value – essential general structures of our acts of judging and asserting. These acts are taken as central – in accordance with the context principle. Thought, truth-value, sense, meaning, are immanent, interdependent aspects of our cognitive activity. On this sort of view, the theory of sense and meaning, in providing the formal demonstration above that the inference “ $a=b, Fa, \text{ therefore } Fb$ ” need not be a mere case of repetition of the premise in the conclusion, makes clear the logical place which identity judgments can occupy. It does not provide a demonstration that this place is filled. *We* fill that place by taking identity as a primitive sign whose sense and meaning we grasp in employing it in accordance with the principle of substitution that governs it.

Thus we should not view the theory of sense and meaning as explaining the phenomena governing the inferential interrelations of identity judgments. Nor should we, however, treat inferential relations among thought contents as primary, expecting this to secure for us a world of objects. Rather, we should simply take it that to speak of inferential relations involving identity statements, and to speak of the objects which we mean and the senses through which we

mean them, are different ways of saying the same things. This is the ultimate resolution of the problem of identity. While Frege pointed the way to such a resolution, however, he never managed to follow it out systematically.

VIII: Problems and Prospects

Frege claims at times that the theory of sense and meaning provides an *explanatory account* of important features of thought and language. In a letter to Jourdain, he writes that “the possibility of our understanding sentences which we have never heard before rests evidently on this, that we construct the sense of a sentence out of parts that correspond to the words. ... Without this, language in the proper sense would be impossible.” (Frege to Jourdain, 1914, *WB*, 127/*PMC*, 79.) Similarly, in correspondence with Peano, he seeks to “*explain* how it is possible that identity should have a higher cognitive value than a mere instance of the principle of identity,” and adds that “my distinction between sense and meaning comes in in an illuminating way.” (Frege to Peano, undated, *WB*, 196/*PMC*, 127, my emphasis.)

Frege’s explanation of the possibility of understanding new sentences is spelled out in his unpublished “Logic in Mathematics” of 1914 (*NS*, 243/*PW*, 225):

It is marvellous what language achieves. By means of a few sounds and combinations of sounds it is able to express a vast number of thoughts, including ones which have never before been grasped or expressed by a human being. What makes these achievements possible? The fact that thoughts are constructed out of building-blocks (*Gedankenbausteinen*). And these building-blocks correspond to groups of sounds out of which the sentence which expresses the thought is built, so that the construction of the sentence out of its parts corresponds to the construction of the thought out of its parts.

And as we take a thought to be the sense of a sentence, so we may call a part of a thought the sense of that part of the sentence which corresponds to it.⁶¹

Frege appeals here to his principle of compositionality for senses, made explicit in the last sentence. However, he puts a particular construction on that principle, shown in his metaphor of “building blocks.” This metaphor suggests a conception of the parts out of which the thought is “constructed,” as independent of and prior to that thought. The context principle however, requires a different conception of the relation of part to whole, according to which, as Aristotle puts it, “a whole is necessarily prior to its parts.” (*Politics*, 1253a20.) Thus, the drive for an explanatory use of the sense-meaning distinction leads away from the context principle here.

One possible motivation for the attempt to provide an explanatory account of the difference in inference potential between “ $a=a$ ” and “ $a=b$ ” is the worry that if we fail to do so, we will be left with a picture of thoughts as individuated solely by their logical interconnections, and so with a coherentism which cannot explain the way in which truth is independent of sense. Purely on the basis of the inferential interconnection of thoughts we cannot find a conception of these thoughts as being determinately *about* specific objects. By treating name-sense as the “mode of presentation” of an *object*, we might hope to avoid such a coherentism and establish an independent link between language and the world. In this way our model of sense will capture the normative dimension of the responsibility of our judgments and assertions to the world, of *truth*. The *BS* model dealt with this by building the objects about which we judge directly into the contents of judgment, but failed to respect the inferential proprieties governing the contents of our judgments. The new model retains an intimate link between the thought which we judge true or false and the objects about which we think, without threatening to disturb the facts about

inference. In this scheme, however, the link to the world becomes explanatorily fundamental – name-sense, in presenting us with objects, is the underlying phenomenon which explains our capacity to judge in truth-evaluable ways; and truth-evaluability becomes the fundamental phenomenon in terms of which norms of inferential propriety are to be explained.⁶²

Once again, however, the drive to put the sense-meaning distinction to explanatory work leads to a certain backing away from the context principle. Names function in language by giving us objects, and although every object must be given in a particular way, it is not obvious that this involves any link to judgment or thought. As *GL* would lead us to predict, this involves Frege in a residual form of psychologism. When we seek to found the notions of sense and thought on the idea of “modes of presentation,” taking this to be prior to and explanatory of the use of names in a system of inferentially interrelated sentences, these “modes of presentation” become hard to distinguish from the psychological representations or ideas (*Vorstellungen*) accompanying the use of words. The worry then arises that an object is, after all, given to different individuals in different ways, even if those individuals use the same word for that object.⁶³

Paradoxically, this psychologizing of sense arises in part from a move designed to secure the objectivity of thoughts. Frege tended to see the the shareability and communicability of thoughts as intelligible only on a model in which thoughts are transcendent entities independent of all human activities of judging, asserting or thinking. In the “Logic” of the 1880’s this idea is already prefigured: “What is true is true independently of the person who recognizes it to be true. What is true is *therefore* not a product of a mental process of inner act; for the product of one person’s mind is not that of another’s...” (*NS*, 3/*PW*, 3, my emphasis.) Frege’s favored metaphor

for understanding, “grasping” thoughts, also emphasizes this independence: “What is grasped, taken hold of, is already there and all we do is take possession of it.” (“Logic” (1897), *NS*, 149/*PW*, 137.) In “Thoughts” (1918-19) he speaks of thoughts as occupying a “third realm” distinct from the external world of things and the internal world of ideas.⁶⁴ (“Thoughts,” 69.)

Yet this conception of thoughts as existing in magnificent isolation from human activities of judging, speaking and acting in the world raises the worry that these same thoughts might have nothing to do with the world of objects. Hypostasized thoughts may stand in ideal logical relations to one another, but our capacity to recognize them as true or false is rendered mysterious by this picture. The idea that “modes of presentation” of objects are *constituents* of thoughts can seem to provide the needed explanation and grounding of the truth-evaluability of thoughts. But modes of presentation, ways of being given, inevitably are modes of presentation *to us*, ways of being given *to us*. Taken as explanatorily basic they cannot be in turn elucidated in terms of inferential relations between the thoughts in which they occur, and our conception of them, if it is to have any content, drifts in a psychologistic direction. Thus we end up with a view of sense involving a curious mismatch between sentence-senses, substantial “thoughts,” objects occupying a “third realm” of their own, and name-senses, insubstantial “modes of presentation,” which are either nothing at all in their own right, or become reduced to occupants of the inner world of ideas. Frege is driven on the one hand to make thoughts, and so senses, into something too objective, and the other hand, senses, and so thoughts, into something too subjective.

Thus Frege’s *psychologizing* of sense shows itself in his wavering over the *objectivity* of sense. On the official view, each sign in a language has a sense which is objectively determined by the structure of the language and which is shareable by all who have a sufficient mastery of

the language. The sense of an expression is contrasted with the representation associated with it: the latter is private, subjective, and can vary from speaker to speaker, whereas the former “may be the common property of many people, and so is not a part or a mode of the individual mind.” Frege argues that “one can hardly deny that mankind has a common store of thoughts which is transmitted from one generation to another. In the light of this one need have no scruples in speaking simply of *the* sense.” (*S&B*, 29.) Yet he takes away with one hand that which he gives with the other. In a much-discussed footnote to the passage in which he holds that anyone who has mastered the language can grasp the sense of a word, he admits that “opinions as to the sense may differ” (*S&B*, 27), and in the same paragraph in which he claims that we may speak of “*the* sense” he adds that “It might perhaps be said: Just as one man connects this idea, and another that idea, with the same word, so also one man can associate this sense and another that sense.” His response is surprisingly weak: “But there still remains a difference in the mode of connection. They are not prevented from grasping the same sense; but they cannot have the same idea.” Here he appeals to the objectivity of sense as an *entity*, but does not try to defend its objectivity as a *feature of language*. But both forms of objectivity are needed if the fact that a “common store of thoughts” is transmitted from generation to generation is to be intelligible. The idea of a “common store of thoughts” is threatened when the sense of our words is reduced to the interpretation placed on them by each individual speaker.

In his late essay, “Thoughts” (*Der Gedanke*), both the subjectivizing, psychologizing aspect of Frege’s thought about sense, and the objectivizing, hypostasizing aspect are on display. Frege first explains a thought as “something for which the question of truth can arise.” (“Thoughts,” 60.)⁶⁵ He then takes up the expression of thoughts by sentences (“Thoughts,” 62-3),

and particularly of the fact that the thought expressed by a given use of a sentence may depend on the circumstances in which the sentence is uttered, as in the case of sentences involving indexicals like “today,” “yesterday,” “here,” or “there.” Frege mentions specifically the case of the word first person singular pronoun: “The same utterance containing the word ‘I’ in the mouths of different men will express different thoughts of which some may be true, others false.” (“Thoughts,” 64.) He adds that “the occurrence of the word ‘I’ in a sentence gives rise to further questions.” (“Thoughts,” 65.)

He considers an example: “Dr. Gustav Lauben says, ‘I was wounded’, Leo Peter hears this and remarks some days later, ‘Dr. Gustav Lauben was wounded’. Does this thought express the same thought as the one Dr. Lauben uttered himself?” Frege moves from this to a discussion of proper names, remarking that “knowledge of the language is a special thing when proper names are involved.” He suggests that different people will express different thoughts using the sentence “Dr. Lauben was wounded,” insofar as they possess different identifying information about Dr. Lauben and so associate different senses with the name “Dr. Lauben.” In such cases, he says, “as far as the proper name ‘Dr. Gustav Lauben’ is concerned,” they “do not speak the same language.” This is due to the fact that “with a proper name, it is a matter of the way that the object so designated is presented. This may happen in different ways, and to every such way there corresponds a special sense of a sentence containing the proper name.” It is only through a *stipulation* on our part that we can assure “that for every proper name there shall be just one associated manner of presentation of the object so designated.” (“Thoughts,” 65-66.) He remarks that there are occasions on which it is important that such a stipulation be fulfilled.

Yet it is hard to see how to guarantee the fulfillment of such a stipulation, given his

preceding argument. That argument turned on the thought that speakers possessing differing knowledge of Dr. Lauben would associate different modes of presentation with the name “Dr. Lauben.” Here the sense, as mode of presentation, already takes on a highly subjective cast, and it is hard to avoid the idea that each individual might associate his or her own sense with each word, so that the public language of communication would split into a multiplicity of idiolects. When Frege returns to “I” this thought is reinforced (“Thoughts,” 66):

Now everyone is presented to himself in a special and primitive way, in which he is presented to no-one else. So, when Dr. Lauben has the thought that he is wounded, he will probably be basing it on this primitive way in which he is presented to himself. And only Dr. Lauben himself can grasp thoughts specified in this way.

Here, Frege admits that there may be modes of presentation, and so thoughts, which are unshareable and incommunicable. With this, it is hard to see any difference remaining between “mode of presentation” and subjective “idea” or “representation” (*Vorstellung*). The psychologizing of sense is complete. Even though Frege goes on to say that Dr. Lauben *can* communicate a closely associated thought, using “I” in the publicly accessible sense of “the speaker,” once an incommunicable sense has been let in, the fact (if it is a fact) that some senses *are* shareable becomes a mysterious and contingent feature of human psychology.

Frege recognizes that there is a problem here. Immediately after this discussion he raises “a doubt:” “Is it at all the same thought which first that man expresses and then this one?” (“Thoughts,” 66.) In response to this, he argues for a distinction between thoughts and subjective representations (*Vorstellungen*). His ground for this is that “representations are something we have,” which “need an owner,” and have “only one owner,” whereas “other people can assent to

the thought that I express,” so that “I am not its owner.” This shareability and communicability of thoughts is equally required if there is to be any possibility of real disagreement or dispute. (“Thoughts,” 66-9.) At this point, Frege introduces the idea of a “third realm,” in which thoughts are to reside. I have to “acknowledge thoughts as independent of me” if I am to acknowledge that “others can grasp them just as much as I” and so also to acknowledge “a science in which many can be engaged in research.” (“Thoughts,” 74.) Unlike a representation, “we do not *have* a thought;” rather we grasp a thought in thinking, but the thinker “is the owner of the thinking, not the thought.” (“Thoughts,” 74-5.) Frege’s argument here emphasizes the objectivity of thoughts as necessary for the explanation of shared human intellectual activity. Yet this argument is too little, too late; it really bypasses the doubt it was supposed to address, namely how this is *possible*, if thoughts are *made up out of* apparently subjective modes of presentation of objects.

The conception of thoughts, the items which we recognize as true in judging, as transcendent entities occupying a third realm, attempts to ground the objectivity of the norms which govern our acts of judging in a set of mythical objects. Yet these thought-objects have to bear a double explanatory burden, since our judgments are appraised normatively both in terms of their relation to other judgments and in terms of their relation to the world. We may feel forced to choose a direction of explanatory priority – either we take sentences, thought, judgment, truth and inference as explanatorily prior, or we take naming, objects and the ways in which they are given as prior. On the first option we take heed of the context principle and focus on the third realm of logically interconnected thoughts. But we risk coherentism – we are unable to provide a satisfactory account of the independence of truth from inferential articulation. On the second option, we try to secure this independence through appeal to our being *given* a shared

world of objects, of meaning; but this ends up, through neglect of the context principle, plunging us into psychologism and so undercutting any satisfying account of logical norms. Frege's thought exhibits fundamental tensions which can be explained as aspects of an oscillation between these two options. I will conclude by examining two such tensions, one having to do with our grasp of thoughts and senses, the other with the relation of sense to meaning.

Frege's conception of thoughts as *objects* is implicit in his attempt in *BS* to state criteria of identity for judgeable contents, when taken together with *GL*'s account of name-hood, and so object-hood, in terms of occurrence in identity contexts. Thus, in *BS*, sentences can flank the identity sign, and judgeable contents can be "determined in different ways." This would suggest a conception of modes of presentation of *judgeable contents*. Dummett argues on this basis against the idea that *thoughts*, like the early "judgeable contents," are to be individuated in terms of their consequences. Dummett calls this the "map-reference view of language" (*IFP*, 43-45), and complains that this view "demands the introduction of a third feature of sentences, one which stands to the sense of a sentence as sense stands to reference." Thus we would be forced "to acknowledge a third feature of sentences, which we might call their *significance*: the significance of a sentence would consist in the particular manner in which it indicated a specific thought." For Dummett this is fundamentally anti-Fregean:

the whole point of Frege's notion of sense is that there is no place for such a conception...

A proper name, for example, stands for an object; and the particular manner in which it does this *is* its sense... there is no room for a further notion of the particular way in which the sense of the name is picked out, because everything that belongs to the manner in which the expression functions to determine a referent is part of its sense. And what goes

for a complex proper name also goes for a sentence. The sentence stands for its truth-value; and the particular manner in which it determines one or other of the two truth-values *is* the thought it expresses.

Dummett conceives of the core notion of sense as that of mode of presentation, and concludes that as a thought is the sense of a sentence and a truth-value is its meaning, a thought must be a mode of presentation of a truth-value. However, Frege never says this. On the other hand, in the unpublished “Logic” of 1897 he comes very close to the conception of significance which Dummett castigates as clearly “not Frege’s.”

Under the heading “Separating a thought from its trappings” Frege discusses ways in which the same thought can be presented linguistically in different forms, using the familiar example from *BS* of active and passive voice, as well as the example from *S&B* of “*p*” and “it is true that *p*.” He then goes on to say:

The distinction between what is part of the thought expressed in a sentence and what only gets attached to the thought is of the greatest importance for logic. The purity of the object of one’s investigation is not of importance only to the chemist. How would the chemist be able to recognize, beyond any doubt, that he has arrived at the same results by different means, if the apparent difference of means could be traced back to impurities in the substances used? There is no doubt that the first and most important discoveries in a science are often a matter of recognizing something as the same again (*Wiederkennungen*). However self-evident it may seem to us that it is the same sun which went down yesterday and rose today, and however insignificant this discovery may seem to us, it has certainly been one of the most important in astronomy and perhaps the one

that really laid the foundations of the science. It was also important to recognize that the morning star is the same as the evening star, that three times five is the same as five times three.

Here he has segued from the discussion of thoughts expressed in different verbal forms to the discussion of objects given in different ways. His reference to “*Wiedererkennungen*” echoes *GL*’s description of numbers as “self-subsistent objects that can be recognized as the same again” (*selbständige, wiedererkennbare Gegenstände*) (*GL*, 68), and *S&B*’s example of the “re-identification (*Wiedererkennung*) of a small planet or comet” (*S&B*, 25), and even his example of the rising sun is borrowed from *S&B*’s opening paragraph. He concludes that in logic “the first and most important task is to set out clearly what the objects to be investigated are. Only if we do this shall we be able to recognize the same as the same: in logic too, such acts of recognition probably constitute the fundamental discoveries.”⁶⁶ (*NS*, 152-154/*PW*, 141-143.) Frege doesn’t speak explicitly of “modes of presentation of thoughts,” but talk of thoughts as “objects” which have to be “recognized as the same” clearly suggests the idea. Dummett is right to see a tension here, but it is a tension within Frege’s thought, not between somebody’s misunderstanding of Frege and Frege. This tension arises from a potential mismatch between the two notions which generate the level of sense: the notion of mode of presentation, and the notion of inferentially articulated content. This mismatch becomes serious when we hypostasize senses as entities.

This potential mismatch is also evident in a related view of Frege’s which Dummett sees as in need of revision: Frege’s theory of indirect sense. As discussed above, Frege held that in indirect discourse contexts, words have as their “indirect meaning” their customary sense. He

further held that in such contexts words have an “indirect sense,” presumably a mode of presentation of their customary sense. Here, though, we have exactly the notion of a way in which the sense of an expression is picked out, for which Dummett said there was no room in Frege’s account. Dummett therefore proposed a revision of Frege’s account: in indirect discourse contexts the sense of a word remains its customary sense; it is only its meaning which is changed. (*FPL*, 266-269.) That this is a real revision in Frege’s view is shown by his appeal to the doctrine of indirect senses (and indeed the whole hierarchy of doubly indirect senses, triply indirect senses and so on) in dismissing a paradox of Russell about classes of propositions. (Frege to Russell, 28 December, 1902, *WB*, 236-237/*PMC*, 153-154.) In this discussion Frege shows that his conception of thoughts as occupants of a “third realm” is at the root of the idea of indirect sense – thoughts, as *objects*, can be given in different ways just as much as numbers can. Dummett’s worries about this notion have their source in the dubiousness of the idea that “modes of presentation” can be constituents of *objects* which can be “given in more than one way.”⁶⁷

At the same time, Frege’s conception of thoughts as objects, combined with the thesis that word-senses are the “building blocks” out of which thoughts are constructed, results in a hypostasization of name-senses which opens up the question of the representational link between such senses and their meanings, a question which is out of place when name-senses are thought of as “modes of being given” the meaning. The intelligibility of this question entails that there can be sense without meaning, and so thoughts without truth-value. Yet, as Dummett (“Truth,” *FPL*, *IFP*) and Ricketts (“Generality, Meaning and Sense”) have pointed out, Frege’s commitment to the possibility of sense without meaning brings with it an ineliminable use of the

truth-predicate and undercuts the tenability of the claim that “ p ,” “‘ p ’ is true,” and “the thought that p is true” are everywhere interchangeable. For if “ p ” is a sentence containing a name which has sense but no meaning, “ p ” expresses a thought, but has no truth-value. In this case, therefore, “ p ” is neither true nor false, but “‘ p ’ is true” and “the thought that p is true” are false. Here “true” and “false” appear to function as genuine predicates. This is not a merely inconvenient consequence of Frege’s acceptance of sense without meaning; it undercuts the very basis of his account of the “peculiar and incomparable” act of judgment by reintroducing the notion of truth as a property of thoughts.

Ricketts sees this problem as arising from Frege’s theory of generality. (“Generality,” 184-185.) He points out that Frege’s account of the generality of a universally quantified sentence amounts to this: “ $\forall x\Phi x$ ” implies all its instances, all sentences of the form “ Φn ” where “ n ” is a name. Hence, if we deny any instance “ Φn ” we will be committed to denying the general statement “ $\forall x\Phi x$.” However, this poses a serious problem when we admit descriptions of the form “The Ψ ” as proper names, as Frege does. For supposing, as is plausible, that if “ $\Phi(\text{The } \Psi)$ ” is true, there is exactly one Ψ , then whenever this condition is not met, we cannot affirm “ $\Phi(\text{The } \Psi)$ ” – so it would seem that we should deny it. But this means that we will have to deny the generalization “ $\forall x\Phi x$ ” as well, even if this is a logical truth like “ $\forall x(x=x)$ ”.

Frege’s solution to this problem is to deny that “ $\Phi(\text{The } \Psi)$ ” *implies* that there is exactly one Ψ ; this is not a consequence of the first claim, but a *presupposition* of our use of the name “The Ψ .” When this presupposition fails, the name “The Ψ ” has no meaning, and the entire sentence “ $\Phi(\text{The } \Psi)$ ” lacks a truth-value. We do not *assert* that $\Phi(\text{The } \Psi)$, but we do not *deny* it either, and so the account of generality is preserved; but in saving the account of generality, we

are forced to admit that there can be sense without meaning, thought without truth-value.

Ricketts' diagnosis of the situation is persuasive but incomplete.⁶⁸ Frege applies his doctrine of presupposition not only to definite descriptions but to genuine proper names: "that the name 'Kepler' designates something is just as much a presupposition for the assertion

Kepler died in misery

as for the contrary assertion." (S&B, 40.) Ricketts' argument will not provide a clean explanation of this case unless we suppose that Frege held a description theory of the sense of all proper names. If we try to run Ricketts' explanation directly, we will have to put, in place of the claim that there is exactly one Ψ , the explicitly metalinguistic claim which Frege introduces:

"Kepler" designates something, *i.e.* $\exists x(\text{"Kepler" designates } x)$.

Here *designates* appears as a relation between "Kepler" and an object. However, we have to ask whether we are speaking of "Kepler" as a *figure*, or as a *sign*, a figure in *use*. Is the *presupposition* of the assertion that Kepler died in misery, really (1) $\exists x(\text{"Kepler" designates } x)$, or (2) $\exists x(*\text{Kepler* designates } x)$? In the first case, we have employed a "designation relation" between a sign-in-use and its meaning. But as I argue above, the doctrine that truth is not a property of thoughts implies that there is no such relation. To admit such a relation is precisely to admit as well a real property of truth for *sentential signs*, and so also for the thoughts which those signs essentially express. If we do not admit such a relation, I have argued, then to assert that "*n*" designates *x* is simply to assert that $n=x$. In that case, the supposed presupposition would reduce to the bare logical truth " $\exists x(\text{Kepler}=x)$."

On the other hand, in the second case, we have only a designation relation between *figures* and objects. But, even if there is no designation relation between *signs* and meanings, or

between name-senses and meanings, there is surely no reason for Frege to deny that there is such a relation between *figures* and meanings. For it is after all an empirical fact that we use the figure *Kepler* to designate Kepler; this point was central to the opening argument of *S&B*. As in the case of the truth-predicate for figures discussed above, though, the proper analysis of the designation relation for figures is:

a designates *a* = *a* expresses •a• and •a• designates *a*.

Here given that senses essentially designate their meanings, the second clause adds nothing, and all the interesting work goes into the expression relation between figures and name-senses. But what is important for our purposes is that the admission of such a relation does not bring with it a need for sense without meaning. To deny that *Odysseus* designates something in *this* sense, or to claim that *Odysseus was set ashore at Ithaca while sound asleep* is neither true nor false in *this* sense, need not imply that we have sense without meaning, or a truth-valueless thought. For after all no such conclusion follows from the claim that *Suessydo* fails to designate something, or that *Suessydo saw tes erohsa ta Acahti elihw dnuos peelsa* lacks a truth-value.

Thus, if we understand Frege's doctrine of presupposition along the lines of (2), we can defend his account of generality without having to accept truth as a real property of thoughts or designation as a real relation between name-senses and objects. Frege, however would object to this solution; for it would require assimilating the case of *Odysseus* to that of *Suessydo* – both would have to lack not only meaning, but also sense. And yet, isn't it *obvious* that the two cases differ in that we *understand* the sentence "Odysseus was set ashore at Ithaca sound asleep"? Don't we grasp in this a thought, which will "remain the same whether 'Odysseus' means something or not"? (*S&B*, 33.)

To this, the proper answer is that *if* by “grasping a thought” we mean an act which is preparatory to *judging*, recognizing as true, then if *Odysseus* lacks a meaning, we have *not* here “grasped a thought.” Our inner experiences may be indistinguishable in this case from those which we have when grasping a thought, but to suppose that *this* guarantees that we *have* grasped a thought is to slide once again into a form of psychologism. In thus taking sense to be transparent to the “grasping” mind, we threaten to reduce it to the level of a mere mental “representation.” We must accept that we can be under the *illusion* of having grasped a thought, and so of being in a position to ask a question which we can proceed to answer.

Jim Conant, in “The Search for Logically Alien Thought,” has pointed out that Wittgenstein, in the *Tractatus*, saw clearly the need to repudiate this residual element of psychologism in Frege’s thought: (*TLP*, 5.4733)

Frege says: Every legitimately constructed proposition must have a sense; and I say:

Every possible proposition is legitimately constructed, and if it has no sense this can only be because we have given no meaning to some of its constituent parts.

(Even if we believe that we have done so.)

It is a tribute to the “great works of Frege” (*TLP*, Preface) that even if they did not reach fully to this *Tractarian* insight, they were capable of inspiring it.

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NOTES

1. I avoid detailed discussions of the secondary literature. A few references will be given in footnotes, but it will be obvious to many that my debts are far more wide-ranging than can be acknowledged here. The bibliography lists the works from which I have drawn in the course of composing this essay. The most important overall influences on my interpretation are Brandom, Dummett, McDowell, Ricketts, Sluga, and Weiner. Burge on truth, Mendelsohn on identity and Taschek on sense are each of central importance at specific points. A detailed and rich recent reading of Frege, with many points of contact with my interpretation, is that of Michael Beaney. Since the composition of the main argument of this essay, several relevant works have appeared of which it has not been possible to take account. The bibliography lists these works as well.

I first studied Frege under Bob Brandom at the University of Pittsburgh. I sometimes think that everything I say about Frege is a dim recollection of something Bob said in a lecture. I am also in the debt of the many students who have tolerated my spinning these tales in my own lectures. Finally, thanks are due to Jim Conant, Mike Beaney, Gottfried Gabriel, Ed Zalta, Marian David, and especially Tom Ricketts for helpful comments and discussion.

2. The so-called *Jäsche Logik* of 1800 is often held to be of dubious value in interpreting Kant. It was assembled out of Kant's lecture notes by Benjamin Jäsche, with Kant's approval. However, it was known in the 19th century as "Kant's Logic," and included in Kant's collected works. Frege cites it as representing Kant's views. I follow Frege here, since my intention is not to interpret Kant but to illuminate the development of Frege's thought.

3. The importance of Frege's early notion of conceptual content was first made clear to me in Bob Brandom's 1982 Frege seminar. Beaney also emphasizes the *BS* account of sameness of

content. However, he wrongly claims this to be a major innovation on Frege's part. (*FMS*, 56-64.) As we have seen, the fundamental idea is already present in Kant's *Logic*.

Dummett questions the value of taking seriously Frege's introduction of "content" in *BS*, arguing that it is "superfluous to credit him with some rival theory" on the "exceedingly thin" basis of *BS*, §3. (*IFP*, 298, 301.) However, a number of otherwise mysterious claims in Frege's early works become clear when "content" is understood in the manner sketched here: (1) Frege's appeals to the inverse proportionality principle; (2) his claim, discussed below, that the axioms of the *BS* "have enough content" since they are "adequate to the task" of proofs; and (3) his argument that his choice of logical primitives is superior to Boole's, since his primitives have a *simpler content*. He claims that "the simpler a content is, the less it says" and that the material conditional is simpler than Boolean identity (material biconditional), conjunction, or exclusive disjunction. He sees Jevons' replacement of exclusive with inclusive disjunction as an "improvement" because it "diminishes the content of the sign." Similarly, vein he speaks of adding "an unnecessary condition to a judgement" as resulting in a "diminution of content." ("Boole's logical Calculus and the Concept-script," *NS*, 40-1, 43/*PW*, 36, 38.) All of this makes sense if we think of content in terms of consequences – the fewer the consequences, the less the content.

4. The development of Frege's conception of judgment, and its relationship to Kant's, is discussed further in my "Judgment and Truth in Frege."

5. Gentzen took consequence to be a relation between *sequences* of premisses and conclusions, allowing "multiple conclusions." Here, the premisses are taken as a set, and there is only one conclusion of any inference.

6. Beaney (*FMS*, 57) and Mendelsohn (“Frege’s *Begriffsschrift* Theory of Identity,” 287) argue that **(B)** follows from **(CONTENT)** and identity; but from that principle, we can only conclude that sentences with the same content imply each other, but *not* conversely. Beaney thinks that **(A)** then follows from **(B)** (*FMS*, 63), but this requires further assumptions about logical truth.
7. One could also get the problematic result **(A)** from this principle directly.
8. This aspect of Frege’s conception of logic is emphasized in Detlefsen’s “Fregean Hierarchies” and in Burge’s “Frege on Knowing the Foundations.”
9. A similar, but slightly more complicated case, occurs in the first sequence of proofs in *BS*, culminating in (5), $(b \supset a) \supset ((c \supset b) \supset (c \supset a))$. To prove this, Frege begins with axiom (2), $(c \supset (b \supset a)) \supset ((c \supset b) \supset (c \supset a))$. He uses an instance of axiom (1) to “weaken” this to (3), $(b \supset a) \supset [(c \supset (b \supset a)) \supset ((c \supset b) \supset (c \supset a))]$, which he combines with an instance of (2) to obtain (4), $[(b \supset a) \supset (c \supset (b \supset a))] \supset [(b \supset a) \supset ((c \supset b) \supset (c \supset a))]$. (5) follows from (4) and an instance of (1). Here the “diminution of content” occurs as a “necessary transition point” in the move from (2) to (3). (Here and elsewhere I modernize Frege’s notation silently.)
10. See the second rule of *GG*, “interchange of subcomponents.” (*GG I* §47, 61.)
11. Hence there is no need to introduce, in addition to Frege “conceptual content,” Beaney’s epistemologically motivated notion of “cognitive content.” (*FMS*, 64.)
12. My account of Frege’s notion of content might seem to depend on a non-Fregean conception of logic, since it involves a consequence relation that can hold even when premisses and conclusion are not truths. For Frege’s considered view took valid inference to move from true premisses to true conclusions. (“Foundations of Geometry: Second Series, III, 426.)

However, it can be argued that Frege only came to such a conception of logic *after*

drawing the sense-meaning distinction. In some notes on Lotze's *Logik*, which Franz Hovens convincingly dates to the early 1880's (Hovens, "Lotze and Frege"), Frege wrote that "the task of logic is to set up laws according to which a judgment is justified by others, irrespective of whether these are themselves true." ("17 Key Sentences on Logic," *NS*, 190/*PW*, 175.) In contrast, shortly after *S&B*, he criticized "content logicians" (*Inhaltslogiker*) for forgetting "that logic is not concerned with how thoughts, regardless of truth-value, follow from other thoughts." ("Comments on Sense and Meaning" (1892-5), *NS*, 133/*PW*, 122.) It seems that it was the sense-meaning distinction that forced on Frege the question of whether logic is primarily concerned with the realm of meaning or the realm of sense. It is a further issue why he chose to answer this question by asserting that "the laws of logic are first and foremost laws in the realm of meanings and relate only indirectly to sense." (*NS*, 133/*PW*, 122.) This is not the place for a developed answer to this question, but I will offer the following speculative suggestion: if logic is to yield us knowledge of *objects*, such as the numbers, it must operate at the level of meaning. However this may be, even in his later period Frege surely recognized that thoughts stand in relations such that were certain thoughts to be true, other thoughts would have to be true – relations specifying "how thoughts, regardless of truth-value, follow from other thoughts." His later view was simply that such relations are not the concern of *logic*.

13. "Boole's logical Calculus and the Concept-Script," 1882 (*NS*, 17/*PW*, 16.):

I only allow the formation of concepts to proceed from judgments. If, that is, you imagine the 2 in the judgeable content

$$2^4 = 16$$

to be replaceable by something else, by (-2) or by 3 say, which may be indicated by

putting an x in the place of the 2:

$$x^4 = 16,$$

the judgeable content is thus split up into a constant and a variable part. The former, regarded in its own right but holding a place open for the latter, gives the concept ‘4th root of 16.’

Grundlagen, 1884 (*GL* §70, 82):

If from a judgeable content which deals with an object a and an object b we subtract a and b , we obtain as a remainder a relation-concept which is, accordingly incomplete at two points.

14. Compare the passages cited in note 13 above.
15. In *BS*, Frege uses the sign “ \equiv ” for identity; in his later writings he uses “ $=$ ”.
16. This point is also made by Gabriel, “Objektivität.”
17. What follows makes it plausible that in *BS* Frege took double negation to preserve content. There are no other clear examples in *BS*. In “Compound Thoughts” (1923-6), Frege says that “ $\sim\sim A$ ” and “ A ” express the same thought, that “ $A \& B$ ” and “ $B \& A$ ” express the same thought, and that contraposition preserves the thought. (“Compound Thoughts,” 39, 44, 48.)
18. In contrast, the system of the later *GG* has a much stronger identity axiom.
19. See, for example, Mendelsohn, “Frege’s *Begriffsschrift* Theory of Identity,” Weiner, “Frege and the Linguistic Turn,” and Dummett, *IFP*.
20. There is also trouble for quantification into identity contexts, especially into “mixed” contexts; Frege needs to be able to quantify into these contexts, for example in his definition of “many-one procedure” (proposition (115)) and the subsequent theorems.
21. For an illuminating discussion of this see Tappenden, “Extending Knowledge.”

22. This is the theory that mathematical existence is simply consistency. Frege's point is that consistency is a property of the concept with which one defines an object, and does not guarantee existence of the object. The formalist confuses concept and object here. (*GL* §95, 106.)
23. My understanding of the context principle is indebted to Jim Conant's detailed analysis of Wittgenstein's use of it in "The Method of the *Tractatus*."
24. Austin translates "*Vorstellung*" as "idea" in order to bring out this psychological aspect of "*Vorstellung*" in Frege's usage; but this obscures Frege's point in the footnote discussed below.
25. Aware of the fact that "representation" and associated words might be understood in the "objective" sense, Frege repeated such cautions in his later writings. For example, in *S&B* he writes: "We may include with representations intuitions... One may on the other hand understand intuition as including any object in so far as it is sensibly perceptible and spatial." (*S&B*, 29, fn 3.)
26. Higher-order functions are generated by allowing lower-order functions to be the arguments which are omitted and replaced; but recognizing these lower-order functions requires that at some level there are arguments that are themselves not functions.
27. This is also clear from his acceptance of Leibniz' definition of identity (*GL* §65, 76.):
- Things are the same as each other, of which one can be substituted for the other without loss of truth. (*Eadem sunt, quorum unum potest substitui alteri salva veritate.*)
- Beaney sees use-mention confusion here, and assumes that Frege is thinking of substituting *expressions* for one another. (*FMS*, 155.) However, Frege's discussion shows that he is thinking of substituting *objects* within judgeable contents. He writes (*GL* §65, 77.):

In order ... to justify our proposed definition ... we should have to show that it is possible,

if line a is parallel to line b , to substitute

the direction of b

everywhere for

the direction of a .

While Austin and Beaney (*FMS*, 101; *The Frege Reader*, 112) insert quotation marks around “the direction of b ” and “the direction of a ,” these are not present in the German text. A little later Frege remarks that if we introduce any new “assertion about directions” we will need to “make it a rule always to see that it must remain possible to substitute for the direction of any line the direction of any line parallel to it.” Here too identity appears as a relation between objects, not names.

28. Beaney emphasizes the importance of this passage (*FMS*, 102). It is worth noting that, in context, this passage equivocates on the key notion of a “way of being given.” This shows that at this stage Frege had not yet achieved complete clarity on this issue.

The passage occurs as part of Frege’s discussion of a proposed definition of “direction” in terms of parallelism:

the direction of a = the direction of b if and only if $a//b$.

His worry is that while this settles when the direction of one line is identical with that of another, it does not establish a *general* criterion which would settle for any q whether q = the direction of a . Hence, it does not tell us *what* the direction of a line is, does not “give us” the object.

Frege notes that if we had the “concept of direction” we could complete our definition by stipulating that if q is not a direction, $q \neq$ the direction of a . He considers the stipulation

(*) q is a direction if and only if q is introduced by means of the proposed definition.

He replies: (*GL* §67, 78)

If ... we were to adopt this way out, we should have to be presupposing that an object can only be given in one single way; for otherwise it would not follow, from the fact that *q* was not introduced by means of our definitions, that it *could* not have been introduced by means of it.

The passage quoted in the main text follows this immediately.

Yet this is an equivocation. Frege says that (*) presupposes that an object can be given in only one way. This is true *if* he means “a direction can only be given *as a direction*.” He then concludes that if an object can only be given in one way, all identities become sterile instances of the obvious tautology that what is given in the same way is the same. But *this* conclusion follows only if he means that the only way in which the direction of *a* can be given is *as the direction of a*, and this in no way follows from (*). It is fully compatible with (*) that the same object be given as the direction of *a* and as the direction of *b*, in which case the true identity “the direction of *a* = the direction of *b*” will be neither more nor less trivial than “*a//b*”.

The equivocation involved here is between a conception of the “way in which an object is given” which, like Kant’s “form,” permits several objects to be “given in the same way,” and a conception which, like *BS*’s “mode of determination” requires that at most one object can be given (determined) in any particular way. That Frege was capable of this equivocation reveals that his own conception of what it is for an object to be “given” was not yet completely fixed.

29. In a footnote he states “I understand ‘*a=b*’ to have the sense of ‘*a* is the same as *b*’ or ‘*a* and *b* coincide.’” This might be taken to endorse the view that identity is a relation between objects – except that the footnote occurs before the question of the nature of identity is posed.

30. In this discussion I am indebted to William Taschek, “Frege’s Puzzle.”
31. The phrase “refer to the subject matter” translates “*die Sache selbst ... betreffen*,” an echo of *BS*’s “concern the very heart of the matter,” “*das Wesen der Sache selbst betreffen*.”
32. This distinction is orthogonal to Peirce’s type-token distinction; both figures and signs can be either tokens or types. Frege was well aware of such issues, but they need not detain us here.
33. Compare Abraham Lincoln’s answer to the question “If you called a tail a leg, how many legs would a donkey have?” – “Four. Calling a tail a leg doesn’t make it one.” My reference to “possible worlds” is anachronistic but inessential.
34. While in *S&B* he introduces the terminology of “mode of presentation,” after *S&B* he sometimes reverts to talk of “modes of determination.” (Frege to Jourdain, 1914, *WB*, 128/*PMC*, 80; Frege to Russell, 28 December, 1902, *WB*, 234/*PMC*, 153.) The terminology of *S&B*, literally “mode of being given,” “*Art des Gegebenseins*,” harks back to the *GL*’s question, “How are numbers given to us?”
35. The supposed opposition between direct and Fregean theories of reference is not so clear as it is often taken to be.
36. However, he waffles on several of these claims. I discuss the significance of this below.
37. Here, clearly, words must be understood as individuated *as objects* rather than *as signs*.
38. Frege reiterated the possibility of sense without meaning several times in his correspondence with Russell, after the latter’s discovery of the paradox had led him to conclude that “my explanations in sect. 31 [of *GG I*] do not suffice to secure a meaning for my combinations of signs in all cases.” (Frege to Russell, 22 June, 1902, *WB*, 213/*PMC*, 132.) If he had rejected the possibility of sense without meaning, it would have followed that in *GG* he had

neither expressed any thoughts nor carried out any reasoning. Clearly, he may have found this evaluation of his life's work unattractive.

39. This analogy is weak, however. If there *are* senses without meaning, it can hardly be a "fault" to express them. Rather, a language which could not express them would be impoverished.

40. Alasdair MacIntyre made this point to me in conversation.

41. For more on Russell's reaction to Frege see Hylton, "Frege and Russell," and my "The Argument of 'On Denoting'."

42. The following discussion is heavily indebted to Burge, "Frege on Truth," and to the writings of Tom Ricketts, especially "Objectivity and Objecthood."

43. In his later writings, Frege frequently speaks of the "thought-content" (*Gedankeninhalt*) of sentences, and uses "content" and "thought" interchangeably. ("Foundations of Geometry, Second Series, I," 294, among many other examples.) In particular, he often speaks of sentences as having "generality of content" insofar as they have a range of particular sentences as consequences. ("Foundations of Geometry: Second Series, I," 308; again, among many examples.) This phrase, which harks back to his earlier account of judgeable content in terms of consequences, first occurs in his writings in the early 1880's, in "Boole's logical Calculus and the Concept-script." (*NS*, 11, fn ***/*PW*, 11, fn ***)

At the same time, Frege continues to use "content" informally to refer to the meaning, or object named, when speaking of a proper name. Thus, in "On Euclidean Geometry" (1899-1906?), he speaks both of the sense of a sentence as its content, and of the "confusion of numerals and numbers" as an example of the confusion of a "sign and its content." (*NS*, 182/*PW*, 167; again, examples could be multiplied.) Here the continued influence of the *BS* model is

palpable.

44. My presentation follows closely that of Burge, “Truth.”
45. This point is made by Burge, “Frege on Truth.”
46. Compare the criteria for the individuation of thoughts discussed at the end of section I.
47. This argument is repeated by Frege on several occasions; my summary draws on several versions. (Frege to Russell, 28 December, 1902, *WB*, 234-235/*PMC*, 152-153; Frege to Russell, 21 May, 1903, *WB*, 240/*PMC*, 157-158; Frege to Russell, 13 November, 1904, *WB*, 247/*PMC*, 165; “Introduction to Logic” (1906), *NS*, 210-211/*PW*, 194; “Logic in Mathematics” (1914) *NS*, 250-251/*PW*, 232.) The possibility of sense without meaning is inscribed into the heart of Frege’s argument that some sentences have a meaning as well as a sense. Yet this very possibility raises serious difficulties for his theory, as we will see below.
48. Frege also argues for this conclusion on the grounds that it is only the truth-value which is preserved under arbitrary substitutions of names with the same meaning. (*S&B*, 35.) This argument is not convincing, however, as Burge points out – such items as “Russellian propositions” for example, would seem to be preserved under the substitutions in question.
49. In the following I draw especially on the work of Tom Ricketts.
50. There is a much controverted issue concerning whether compositionality for senses conflicts with Frege’s thesis that the same thought can be expressed in radically different ways. I do not myself think there is a conflict – metaphorically, the same thing can be divided into parts in many ways – but I will not discuss this issue here.
51. This is for the simplest case; in more complex cases we may be directed to the concepts which the sentence is about and the higher-level concepts which are applied to them.
52. Ricketts’ “Objectivity and Objecthood” is especially helpful on this argument. See also

his “Logic and Truth.”

53. This point is made by Ricketts in “Objectivity and Objecthood.”
54. Joan Weiner’s work on the “elucidation” of primitive terms is illuminating here.
55. I owe this important point to Burge, “Frege on Truth.”
56. This appears to be the view of Terence Parsons in “What do Quotation Marks Name?”
57. *CP* translates the second occurrence of “*Erkenntniswert*” in this sentence as “for the purpose of acquiring knowledge.” But I think it is important to see that Frege is both linking and contrasting truth-value (*Wahrheitswert*) and cognitive value (*Erkenntniswert*) here.
58. Here again I follow Ricketts, especially “Logic and Truth.”
59. The tightness of the link can be disputed; there is a question whether one can “recognize the truth” of a false thought – judge incorrectly. I assume here that this is possible. In this I disagree with Ricketts (“Logic and Truth”). See my “Judgment and Truth in Frege.”
60. Gottfried Gabriel and Hans Sluga have made clear the connection of “truth-value” to neo-Kantian “value theory.” See Sluga, “Frege on Meaning” and Gabriel, “Frege als Neukantianer.”
61. The translation is Dummett’s (*IFP*, 262).
62. I have neglected a crucial element of Frege’s account here, namely the *concepts* which we apply to the objects about which we think. The unpublished “Comments on Sense and Meaning” establish that Frege intended to apply the sense-meaning distinction to functional expressions such as predicates, as well as names and sentences. (*NS*, 128-136/*PW*, 118-125.) The meaning of a functional expression is a function from argument-meanings to value-meanings, so that in the case of a predicate, the meaning is a concept, or function from objects to truth-value. Any discussion of Frege’s account of the truth of thoughts must take into consideration not only

the objects that the thoughts are about but also the concepts that those thoughts involve. There is a debate, however, over the nature of the *senses* of functional expressions. Dummett takes them to be modes of presentation of functions, while Geach takes them to be functions from argument-senses to value-senses. (See Dummett, *FPL*, and *IFP*; Geach, “Names and Identity;” for an illuminating recent discussion see Sullivan, “The Functional Model of Sentential Complexity.”)

There is much to say on this issue, but space does not permit an extended discussion. I will only offer a brief suggestion here: the two competing interpretations derive from the two sources of the notion of sense – the idea of thoughts as inferentially individuated sentential contents, and the idea of name-senses as modes of presentation of objects. Emphasis on the first idea leads to Geach’s interpretation while emphasis on the second supports Dummett’s. The necessity of seeing these positions as competing and exclusive is questionable however. The appearance of necessity derives from the seeming need to choose one of these ideas as explanatorily more basic. This choice does not have to be forced if we conceive of thoughts and senses as immanent in the activity of judging rather than transcendent Platonic entities.

63. The thought that Frege falls into psychologism in trying to make the notion of name-sense do explanatory work derives from McDowell’s “On the Sense and Reference of a Proper Name.” My discussion is inspired by this thought of McDowell’s as well as by his *Mind and World*.

64. Beaney calls this view of thoughts “semainomenalism.” He sees it as a departure from Frege’s earlier view of contents, “best characterized ... as ‘states of affairs’, constituents of the temporal first realm.” (*FMS*, 213-224, 217.) This reading of the early notion of content reflects the way in which objects figure as parts of such contents. However, the inferential individuation

of such contents points in another direction. The discussion of the objectivity of content in 1880's "Logic" prefigures many of the features of the later explicit conception of a "third realm."

65. Given his account of judgment as "the recognition of the truth of a thought" we can see here the ancestry of the notion of thought in *BS*'s notion of "judgeable content."

66. Similarly, in "Concept and Object," and in correspondence with Husserl, Frege speaks of the necessity of "recognizing a thought as the same again" (*wiederzuerkennen*). ("Concept and Object," 196, fn 7; Frege to Husserl, 9 December, 1906, *WB*, 105/*PMC*, 70.)

67. A similar set of concerns motivates David Bell's discussion in "Thoughts."

68. The argument that follows is inspired by McDowell's discussion in "Truth-Value Gaps."